CELLULAR MANUFACTURING LAYOUT DESIGN USING HEURISTIC CLUSTERING ALGORITHM AND LPP MODEL

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ARTICLE INFO

Article details
Submitted by authors 13 Mar 2020
Accepted for publication 8 Jun 2021
Available online 31 Aug 2021

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DOI
http://dx.doi.org/10.7166/32-2-2340

ABSTRACT

This mathematical model forms machine cells, optimises the costs of unassigned machines and components, and designs the shop floor cell layout to have minimal movement of materials. The complete similarity measure algorithm forms machine cells and part families in a refined form. Later, exceptional elements are eliminated in the optimisation model by using machine duplication and sub-contracting of parts. Then the shop floor layout is designed to have optimised material movements between and within cells. An evaluation of the cell formation algorithm' performance is done on the benchmark problems of various batch sizes to reveal the process's capability compared with other similar methods. The data of machining times are acquired and tabulated in a part incidence matrix, which is used as input for the algorithm. The results from the linear programming optimisation model are that costs are saved, machines are duplicated, parts are sub-contracted, and there are inter- and intra-cellular movements. Finally, the output of the inbound facility design is the floor layout, which has machine cell clusters within the optimised floor area.

OPSOMMING

Hierdie artikel bied 'n wiskundige model aan wat masjien selle skep, die onkostes van nie-toegewysde masjiene en komponente optimeer, en die aanlegvloer uitleg te ontwerp sodat daar minimale beweging van materiaal is. Die volledige eendersheidsmaatstaf algoritme skep masjien selle en onderdeelfamilies in 'n verfynde formaat. Daarna word uitsonderlike elemente van die optimeringsmodel verwyder deur die gebruik van duplikasie en sametrekking van onderdele. Daarna word die aanleg se vloeruitleg ontwerp om minimale beweging van materiaal tussen en binne-in die selle te hê. Die sel skeppingsalgoritme se vertoning word evalueer teen maatstafprobleme van verskeie lot groottes om die proses se vermoë te vergelyk met ander, soortgelyke metodes. Die data van masjineertye word verkry en getabuleer in 'n matriks wat gebruik word as inset vir die algoritme. Die resultate van die lineêre programmering optimeringsmodel is kostes wat gestoor word, masjiene wat gedupliseer word, onderdele wat saamgetrek word tesame met die inter- en intrasellulêre bewegings. Laastens word die aanleg se vloeruitleg bepaal wat masjien sel groeperings binne die geoptimeerde vloer area plaas.

1 INTRODUCTION

Cellular manufacturing (CM) works to increase productivity and production efficiency by reducing processing time, work-in-progress, and material movement time. In a CM system, identical components are formed as families and related machines are formed as cells so that one part family can be manufactured within a machine cell. Among various matrix formulation similarity coefficient methods, such as direct clustering, the bond energy algorithm, rank order clustering approaches, in this paper a complete similarity coefficient method (SCM) is proposed because of its easy cluster analysis and its being more flexible in having production data — for example, a process plan, part demand, and processing time.

The cell formation will have exceptional elements (EE) i.e., exceptional machines and parts. EE creates interactions between two manufacturing cells and void elements (VE) inside block diagonals that affect machine utilisation and grouping efficiency. An exceptional component is a component that requires manufacturing on machines in two or more cells. An exceptional machine manufactures parts from two or more part families. Exceptional elements create inter-cellular movements that affect the independence of the cells and increase costs.

Cell layout design has a direct effect on the function of the manufacturing system through the movement of materials. These can help to increase productivity, reduce work in process and inventories, shorten production lead times, and regulate the flow of materials. The operations of cells and the sizes of cell layouts can vary. The cell layout will have work cells inside it, in which machine tools are arranged in series or cross lines, depending on the process plans for the parts. However, a U-shaped layout can be preferred in a cell design that has simultaneous in-line or cross-movements of materials.

2 LITERATURE REVIEW

In this review, the survey was conducted on recent literature of mathematical approaches using various similarity coefficients. In [1], a new similarity coefficient is incorporated with the job sequence and part volume to form cells alone. Approach [2] preferred to apply Jaccard's similarity coefficient for forming cells alone using the principal component analysis. It [3] was proposed with a new similarity coefficient containing process plans, operation time, demand, and production volume, which are able to form machine cells alone. The job sequence, part volume, inter-cell movement cost, job production cost, and the alternative process plans were [4] used in the proposed similarity coefficient-based heuristic method for CM. The cell formation [5] was done, and also achieved the minimisation of the cumulative cell load variation and cumulative inter-cellular moves. The number of parts, operation sequences, processing time, the capacity of machines, and workload of the machinery were considered as parameters. For the grouping of the equipment, the modified artificial bee colony algorithm was considered. A new multi-objective mathematical model [6] was provided with a consideration of machine reliability and alternative process routes. The literature [7] has provided a new model by considering dynamic production times, and uncertainty demands in designing cells for test problems with real-world dimensions have been solved using simulated annealing and particle swarm algorithms. The review revealed that the similarity coefficient can possess parameters such as job sequence, setup and handling time, changes in demand, and volume, in addition to machining time, machine capacity, and several operations to have the capability to form the perfect machine and part clusters simultaneously without (or only a few) exceptional elements, which can overcome the difficulties faced by other methods.

This section of the review is aimed at evaluating the elimination of costs directly related to the exceptional elements. A paper [8] proposed a nonlinear programming model under potentially dynamic conditions that minimises the cost associated with the estimated demands for inter- or intra-cellular movements of elements (forwards and backwards movements), the existence of exceptional elements, the inter-cellular displacement of machines, and cellular reconfiguration and operational costs and the constant cost of machinery. The modified genetic algorithm [9] for interchanging block diagonal forms is for reducing voids inside cells and exceptional elements outside cells. Two alternative actions are evaluated [10]: a bottleneck machine can be duplicated, or it can be allowed; and manufacturing parameters are incorporated into the proposed simulation study, such as job sequence, batch size, and setup time in comparing the cost of alternatives. Parametric programming [11] is used to reduce the cost of an exceptional element, decrease the number of outer cell operations, and increase utilised machine capacity. The model [12] was proposed to reduce the number of movements between cells, voids, and EEs minimises the total cost of duplicated machines without considering the processing time of operations. A model was [13] proposed that considers reducing the cost of to-and-fro movements between cells and within the cells and the investment cost of machines. The above review revealed that the cost elimination of EE must look into intra-cell and inter-cell movements significantly by correlating machine duplication and part sub-contract respectively.

A layout optimisation model was [14] formulated based on fuzzy demand and machine flexibility, and then developed a genetic algorithm. A systematic methodology [15] was developed to combine the same machines into the cell, which was simulated, and the outcome of the cell design was evaluated by analysis of variance. A split departmental plant layout generation system was described by [16] to develop a facilities layout design that would minimise the material handling costs. Researchers [17] examined how lean production principles had a significant impact on the dimensions of ramp-up factories, including their length and clear interior height. Both the mapping of materials flows and the optimisation of layout [18] were considered through the integration of different operative techniques and commercial software. The

proposed construction-cum-improvement algorithm [19] was a novel combination of analytical steepest descent and heuristic cluster boundary search.

In section 3 of the current work, machine cells are formed by using the proposed similarity coefficient between machines and components, as well as refining clusters by using a weight-based approach in-line, with the similarity measure proposed in a recent model [3]. Finally, a performance evaluation is carried out through machine utilisation (MU), grouping efficiency (GE), and the percentage of exceptional elements (EE). In section 4, a linear programming model is developed to reduce the cost of exceptional elements more than the cost reduction given in [11]. In section 5, the sizes of cells, positioning cells, and machines are determined with the help of the sorting of cells and of machines in relation to their origin; and finally a 2D shop floor layout is designed.

2.1 Notations

A	Aisle between machines or machine to sidewalls
A_i	Cost of machine type i
a_{ij}	1 if part j assigned to machine i, 0 otherwise
a_{xi}	1, if part type x visits machine i; 0 otherwise
a_{xjr}	1 if part type j assigned to machine x with process plan r; 0 otherwise
a_{yi}	1, if part type y assigned to machine j; 0 otherwise
a_{yjr}	1, if part type j assigned to machine y with process plan r; 0 otherwise
B_{ik}	Budget allowed duplicating the bottleneck machine i
BM, BP	Set of pairs of bottleneck machines, bottleneck parts (i, j)
BS_{1x} and BS_{1y}	No. of future batches of part x and y rounded to the whole, D_{1x}/V_x, D_{1y}/V_y
BS_x and BS_y	No. of batches of part x and y rounded to the whole, D_x/V_x, D_y/V_y
C	Number of components
C_i	Periodic capacity of machine type i
C_k	Number of jobs in the k_{th} cell
D_j	Periodic demand for part j
DM	Duplicated machines set connected to exceptional component j
DN, SC	Set of pairs of duplicated machines, sub-contract parts (i, j)
D_x and D_y	demand of part type x and y per period
EE	No. of exceptional elements
GE	Grouping efficiency
i	= 1..., M (machines index)
IAj	Handling cost for a unit of part j within one cell
I_j	Handling cost for a unit of part j between two cells
j	=1…, N (parts index)
k	=1…, N_c (cells index)
M_l	Number of machines along lengthwise of the cell
M	Number of machines
M_k	Number of machines in the k_{th} cell
ML_i	Machine length considered from the dimensions of machine i
M_{rs}/M_{ls}	Number of machines along the right side and left side of the cell
MT_{ij}	Machining time of machine i required for part j
MU	Machine utilisation
MW	Machine width considered as constant for all machines
N	Total number of operations
N_{01}	Total number of operations within the block diagonals
N_c	Number of cells formed
n_j	Number of operations a part j undergoes
n_k	Number of operations in k^{th} cell;
PBs_j	Ratio of no. of batches of part j, BS_j / BS_{1j}
p_j (x_k)	Probability of operations in the j^{th} job
q	Weighing factor $0 \le q \le 1$=0.5
Q_j	Number of machine type i required to process parts in machine cells (integer)
p	= 1.., P (process plans)
R_{ik}	Number of machine type i to be purchased for cell k (integer)
S_j	Subcontracting cost of a part j for a process
sp_{xj}, sp_{yj}	Process sequences of part j in machines x and y respectively
t_{ix}/ C_{tx}	Ratio of processing time which part x in machine i with a cycle time of part x.
t_{iy}/ C_{ty}	Ratio of processing time which part y in machine i with a cycle time of part y.

t_{jxr}/ Ct_x	Ratio of machining time which part j in machine x with a cycle time of part j;
t_{jyr}/ Ct_y	Ratio of machining time which part j in machine y with a cycle time of part j;
UC_{ij}	Usage capacity of machine i for part j (MT_{ij} D_j / C_i)
V_x and V_y	Batch size of part type x and y per period
w_k	Frequency of operations in the k^{th} part family/cell;
X_{ik}, Y_{jk}	1 if machine i and part j occurs in cell k respectively, 0 otherwise
$(n_{oj}\text{-}sp_{xj}+1)/n_{oj}$	Proportion of job sequence weight and no. of operations to be done on part j; this term represents the ratio of process sequence weight of machine x, p_{swxj}
$(n_{oj}\text{-}sp_{yj}+1)/n_{oj}$	Proportion of job sequence weight and no. of operations available on machine y; this term represents the ratio of process sequence weight of machine y, p_{swyi}

3 PRAGMATIC ALGORITHM OF A RESUMPTIVE INLINE (PARI) MODEL

The proposed PARI model forms machine cells and part family clusters in two stages. In the first stage, the similarity coefficient algorithm forms machine cells and part families. In the second stage, as resumption, the same clusters are iterated by a weighting-based method to obtain proper partitioning by refining the machine cells and the part families. The modus operandi of generating high similarity matrices incorporates a job sequence weight ratio, setup and handling time in processing time ratio, the ratio of present and new batch sizes, which have a significant impact on finding complete similarity for machines as well as components. The authors incorporated the necessary manufacturing data, such as alternative process plans and operational sequences, to improve the similarity coefficient in order to process the input data over the other similarity coefficients.

3.1 A complete similarity measure weight-based algorithm

The process plan-weight ratio is calculated between the sequence weight and the number of operations to be done on the part. The ratio of batch size is calculated between the new batch size and the present batch size of parts. The batch size is the ratio between demand and production volume. The use of the time ratio, the sequence weight ratio, and the batch size ratio will help in boosting the similarity between machines as well as parts.

The cycle time of each part is calculated by considering the sum of waiting time, setup time, and handling time, which is assumed to be equal to the machining time for each operation to be performed in the machine.

$$C_{tj} = \sum_{x}^{C}(\mathrm{MTij} + \mathrm{Wij} + \mathrm{STij} + \mathrm{Hij}) \tag{1}$$

From the assumptions of the model, the cycle time is double the value of the machining time of each operation of the part.

$$\sum_{j}^{C} \mathrm{MTij} \simeq \mathrm{Wij} + \mathrm{STij} + \mathrm{Hij} \tag{2}$$

The process sequence weightage ratio is calculated between the weightage of a sequence of the particular operation and the total number of operations to be performed on the part.

$$\mathrm{Pswxi} = (\mathrm{noj} - \mathrm{spxj} + 1)/\mathrm{noj} \tag{3}$$

For machines, compute the similarity coefficient between machines x, y (for forming machine cells) using equation (4), SC_{xy} = 1 if x = y

$$\mathrm{SCxy} = \frac{\sum_{\mathrm{j=1}}^{\mathrm{N}} \sum_{\mathrm{r}\in \mathrm{nx}\,\&\, ny} [\max \left(\frac{\mathrm{Mtjxr}}{\mathrm{Ctj}} \mathrm{X} \frac{\mathrm{noj}-\mathrm{spxj}+1}{\mathrm{noj}}\right) : (\frac{\mathrm{Mtjyr}}{\mathrm{Ctj}} \mathrm{X} \frac{\mathrm{noj}-\mathrm{spyj}+1}{\mathrm{noj}})] \ \mathrm{axjr.\,ayjr.\,PBsj}}{\sum_{\mathrm{j=1}}^{\mathrm{N}} \sum_{\mathrm{r}\in \mathrm{nx}\,\&\, ny} \max[\left(\frac{\mathrm{Mtjxr}}{\mathrm{Ctj}} \mathrm{X} \frac{\mathrm{noj}-\mathrm{spxj}+1}{\mathrm{noj}}\right) \mathrm{axjr} : \ (\frac{\mathrm{Mtjyr}}{\mathrm{Ctj}} \mathrm{X} \frac{\mathrm{noj}-\mathrm{spyj}+1}{\mathrm{noj}}) \ \mathrm{ayjr}] \,.\, \mathrm{PBsj}} \tag{4}$$

The similarity coefficient SC_{xy} is calculated between machines, and the similarity coefficient matrix is formulated from the input part incidence matrix along with the production data, considering all alternative process plans.

For parts, compute the similarity coefficient between parts x, y (for forming part family) using equation (5). SC_{xy} = 1 if x = y;

$$SCxy = \frac{\sum_{i=1}^{M}[\max(\frac{Mtix}{Ctx} X \frac{Bsx}{Bs1x} X\ pswxi : \frac{Mtiy}{Cty} X \frac{Bsy}{Bs1y} X\ pswyi).axi.ayi]}{\sum_{i=1}^{M} max[\frac{Mtix}{Ctx} X \frac{Bsx}{Bs1x}.axi.pswxi : \frac{Mtiy}{Cty} X \frac{Bsy}{Bs1y}.ayi\ .pswyi]} \quad (5)$$

The similarity coefficient of parts is calculated, and a matrix is formulated for each of the process plans. Several parts' similarity coefficient matrices will be equal to several process plans. A good cluster is chosen among various process plans parts matrices.

The quintessential proposal in similarity measure is the inclusion of manufacturing data proportions that have a significant impact on similarity whose purpose is to generate simultaneously perfect clusters of machine cells and part families. The pragmatic algorithm is coded and executed to generate similarity matrices using the C++ programming language (with a 2.13 GHz Core 5 Pentium processor); the production data are given as the input, and the output is obtained as similarity coefficient matrices, and a block diagonal form is obtained through iteration using spreadsheet simulation.

The weights for the rows and columns of the similarity coefficient matrix (machine—machine matrix) are calculated using the equations:

$$\sum_{y=1}^{M} \text{SCxy x X (M} - \text{y) for rows} \quad (6)$$

$$\sum_{x=1}^{M} \text{SCxy x X (M} - \text{x) for columns} \quad (7)$$

Select X > 1, replace M with N for the part—part similarity matrix. Arrange the rows and columns in a descending order of magnitude of the weights obtained.

An iteration using spreadsheet simulation is done until there is no change in the sorting of the rows and columns. Cells and part families are identified from the machine matrix and the part matrix, finally obtaining a block diagonal form from the given machine component incidence matrix.

Clusters are refined sequentially for machine cells and part families, assuming that part family and machine cell formation has already been carried out that is, to use this procedure for machine cell formation when the part family is known, or vice versa. If the components are more than machines, first use the procedure for part family refining; otherwise use the procedure for machine cell refining and then another. A machine cell or part family is formed using the procedure explained earlier, assuming that cell formation has already been done; part family formation is explained here.

Assign weights to each part or a machine to the cell or part family using equations 8 and 9.

$$wk = xk \text{ X } pj\ (xk) \quad (8)$$

where

$$pj\ (xk) = 1\ /\ nj \quad (9)$$

The component or machine is assigned to the machine cell or part family where it has scored the highest weight, and then to the other.

3.2 Numerical illustration

The selected benchmark problems (the literature references to five mathematical approaches and one metaheuristic approach of different sizes are mentioned in each input case study) of small, moderate, and large machine component incidence matrices are solved. The data for part incidence, machining time, job sequence, demand and production volume, and machine capacity are acquired and tabulated, and used as input for generating similarity matrices.

Case study 1: 7 machines X 8 components (input data from [20]) Cells are grouped after four iterations, part families are formed after ten iterations. After the weighting-based approach has been used in a series, a block-diagonal form is obtained.

Case study 2: 10 machines X 8 components. Cells and part families are formed after four iterations in a spreadsheet simulation. After the weighting-based approach has been used in series so that cells are refined, then part families have perfect clusters because there are fewer parts than machines. In this solution, there is only one VE and there are four EEs, as shown in Table 2.

Table 1: Incidence matrix with production data (Input data from [9])

M/P	1	2	3	4	5	6	7	8	Ai	Ci	Bi
1	3(4)				1(3)	1(5)		1(4)	450000	5400	900000
2			3(5)				1(3)		40000	5400	80000
3	2(6)					4(4)		3(3)	800000	5400	1600000
4		1(3)		1(3)					650000	5400	1300000
5		2(4)		3(4)	3(5)				750000	5400	1500000
6			4(4)				3(2)	5(6)	700000	5400	1400000
7	4(3)		2(3)			2(3)		4(3)	610000	5400	1220000
8			1(2)				2(5)		420000	5400	840000
9	1(4)				2(3)	3(4)		2(3)	380000	5400	760000
10		3(5)		2(3)	4(4)				260000	5400	520000
Si	3.5	3.75	3.5	3.25	4.0	4.25	3.75	3.50			
Ii	4.0	4.5	3.5	4.5	4.0	5.0	3.5	4.0			
IAi	4.25	5.0	4.5	5.0	4.5	5.0	4.0	4.5			
Di	275	300	280	335	300	350	350	250			
Vi	35	30	35	40	35	35	35	30			

Table 2: Block diagonal form

M/P	1	6	8	7	3	2	4	5
1	1	1	1					1
3	1	1	1					
7	1	1	1		1			
9	1	1	1					1
2				1	1			
8				1	1			
6			1	1	1			
10						1	1	1
5						1	1	1
4						1	1	

Case study 3: 9 machines X 10 components (input data from [11]). The machine and part similarity matrices are transformed into block diagonal form after three iterations and five iterations respectively, and cells and part families are formed.

Case study 4: 10 machines X 12 components **(**input data from [21]). The machine and part similarity matrices are transformed into block diagonal form in the spreadsheet simulation after three iterations and seven iterations respectively.

Case study 5: 10 machines X 20 components **(**input data from [23]). The incidence matrix is transformed using a similarity coefficient heuristic algorithm; cells are formed after three iterations, part families are also formed after two iterations. After the weighting-based approach used in a series, the relocation of clusters has taken place in both horizontal and vertical directions; finally, the block-diagonal form is obtained.

Case study 6: 18 machines X 24 components **(**input data from [24]). The block diagonal is now formed through iterations in similarity and weight-based methods over the machine and part matrices after seven iterations and ten iterations respectively.

Evaluation through performance criteria for 10 x 8 machines components case study:
Machine utilisation: Machine utilisation is a parameter for measuring the goodness of a solution. It denotes the proportion of time the machines within cells are used in production.

$$MU = N01/\sum_{k=1}^{Nc} Mk \text{ X } Ck \tag{10}$$

N01 = 26; Mk1 = 4 Ck1 = 3; Mk2 = 3; Ck2 = 2; Mk3 = 3, CK3 = 3;
MU is 96%.

Grouping efficiency:

$$GrE = (1\text{-}q) \text{ X } 1\text{-}\left\{ EE/[(M \ X \ C) \sum_{k=1}^{Nc} (MK \text{ X } CK)] \right\} + q \text{ X } MU \tag{11}$$

GrE is 98%.

Percentage of exceptional elements: Several elements that occur out of the diagonal blocks are called exceptional elements, which denote the impact of cell formation. The best cell formation approach ends in a lower percentage of exceptional elements.

$$PE = EE / N \quad (12)$$

For EE = 4, out of 30 (N), PE is 13%; the criteria for other case studies are shown in Table 3.

Table 3: Comparison of results PARI algorithm and other best-known approaches

Case study	Size m x c	No. of cells Nc	Proposed approach results %			Best known results %			Method, literature reference
			MU	GrE	PE	MU	GrE	PE	
1	7 X 8	2	80	80	12.5	88	68	26	Direct cluster algorithm[22]
2	10 X 8	3	96	98	13	86.9	87	23	Heuristic GA approach[9]
3	9 X 10	3	90	68	27	88	67	27	Parametric model [11]
4	10X 12	3	83	82	10.5	83	81	10.5	Complete design model[21]
5	10x20	3	100	100	0	94	100	12	Flow matrix algorithm [13]
6	18x24	3	53.3	65.0	21.5	31.0	53.6	38.6	Correlation analysis[24]

3.3 Discussion of results

This proposed PARI algorithm gives an optimal solution with minimal or no exceptional elements as one of the objectives. The solution of the small and medium-size case studies (1, 2, 3, 4, and 5) has few EEs or VEs. The machine grouping is effective in the large-size case study, and it has fewer EEs and VEs. This proposed similarity coefficient can achieve clear partitioning in block diagonal form, if even the case study is large (Case study 6). So this similarity measure algorithm is suitable for any part volume with no or few exceptional elements.

The small and moderate part machine—component case studies (Case studies 1, 2, 3, and 4) are solved effectively in less iteration time to get better solutions. The large-size machine component case studies (case studies 5 and 6) are solved with more than average performance compared with typical methodologies. EEs are equal in case study 4 with a benchmark case study, but inter-cell movements are fewer, and there are more voids than with the benchmark solution. The block diagonal form is given as input for an optimisation model and facility layout design, and transformed into a shop floor layout design for all the case studies in section 5.

4 OPTIMISATION MODEL

The aim of the proposed linear programming model is to reduce EE costs. The inferences from the literature review are the cost of EE, such as inter- and intra-movements; duplication and sub-contracting make up most of the recent crisis in the manufacturing sector. If duplication were is the only remedy for an exceptional element, then there would be no inter-cellular movements; so it is necessary to include intra-cellular movements in the objective function as the proposal in this work rather than the cost's elimination given by [4]. The other proposal in this model is the inclusion of the budget constraint to set a limit to machine duplication.

4.1 Decision variables

Z_{ijk} - Number of inter-cell movements required by part j when machine i is not available in cell k,
W_{ijk} – Number of intra-cell movements required by part j w.r.t to machine i in cells(s) k,
O_{ijk} - Number of units of part j to be sub-contracted when machine i is not available in cell k,
M_{ijk} - No. of machine i dedicated by duplication to cell k for producing exceptional part j.

Step 1: The objective function is to maximise the sum of the savings by either duplicating the exceptional machines or sub-contracting the exceptional parts in the original cell.

The objective function is to maximise the savings by minimising:

$$\sum_k^c \sum_i^M \sum_j^C (\mathbf{Ai}.\mathbf{Mijk}) + \sum_k^c \sum_i^M \sum_j^C (\mathbf{Ij}.\mathbf{Zijk}.\mathbf{Bsj}) + \sum_k^c \sum_i^M \sum_j^C (\mathbf{Sj}.\mathbf{Oijk}.\mathbf{Dj}) + \sum_k^c \sum_i^M \sum_j^C (\mathbf{IAi}.\mathbf{Wijk}.\mathbf{Bsj}) \quad (13)$$

Equation 13 is an objective function that minimises machine duplication cost, inter-cellular movement cost, parts sub-contracting cost, and intra-cellular movement cost. The input coefficients of the objective function take the individual direct values of the input data for the given period of regular manufacturing operations.

Step 2: The constraints for bottleneck machines, intra-cell movements, and bottleneck parts with respect to sub-contract as well as inter-cell movements originally assigned to the same cell are:

$$\mathrm{Xik} - \mathrm{Yjk} + \mathrm{Uijk} - \mathrm{Vijk} = 0 \quad (14)$$

$$\sum_k^c \sum_i^M \sum_j^C MTij\ x\ Dj \le Ci \quad (15)$$

$$\sum_i^M \sum_j^C \mathrm{Mijk} \le \mathrm{Rik} \quad (16)$$

$$\mathrm{Ci} / (\mathrm{MTij\ x\ Dj}) \ge \mathrm{Qi} \quad (17)$$

$$\sum_k^c \sum_i^M \sum_j^C Mijk\ x\ MTij\ x\ Dj \le Ci \quad (18)$$

$$\sum_k^c \sum_i^M \sum_j^C \begin{matrix} if\ (Xik=1\ \&\&\ Yjk=0 \&\&\ Incij=1) \\ Uijk=1; Vijk=Wijk=0; \end{matrix} \quad (19)$$

$$\sum_k^c \sum_i^M \sum_j^C \begin{matrix} if\ (Xik=0\ \&\&\ Yjk=1 \&\&\ Incij=1) \\ Vijk=1; Uijk=Wijk=0; \end{matrix} \quad (20)$$

$$\sum_k^c \sum_i^M \sum_j^C \begin{matrix} if\ (Xik=1\ \&\&\ Yjk=1) \\ INijk=1; \end{matrix} \quad (21)$$

$$\sum_k^c \sum_i^M \sum_j^C \begin{matrix} if\ (Uijk=1 \&\& Xkj=1 \&\& (B[k][i] \le (D[k][j]*S[k][j]*52*5)) \\ \mathrm{DN}ijk=1;\ \mathrm{SC}ijk=0; \end{matrix} \quad (22)$$

$$\sum_k^c \sum_i^M \sum_j^C \begin{matrix} if\ (\mathrm{V}ijk=1 \&\& Ykj=1 \&\& (B[k][i] > (D[k][j]*S[k][j]*52*5)) \\ \mathrm{DN}ijk=0;\ \mathrm{SC}ijk=1; \end{matrix} \quad (23)$$

$$\sum_k^c \sum_i^M \sum_j^C Mijk == DNijk \quad (24)$$

$$\sum_k^c \sum_i^M \sum_j^C Zijk == SCijk \quad (25)$$

$$\sum_k^c \sum_i^M \sum_j^C Oijk == SCijk \quad (26)$$

$$\sum_k^c \sum_i^M \sum_j^C Wijk == INijk \quad (27)$$

$$\mathrm{Xik,\ Yjk,\ Uijk,\ Vijk,\ INijk,\ DNijk,\ SCijk} = 0\ \mathrm{or}1 \quad (28)$$

$$\mathrm{Rik,\ Qi} = \mathrm{integer} \quad (29)$$

Equation 14 ensures that each machine and component is assigned to one cell only. Constraint 15 ensures that the sum of machining times of operations in each machine is within capacity. Constraint 16 is to check

that the machines to be duplicated in each cell to process the part are fewer than the total number of duplicated machines of the same type in the cell. Constraint 17 is to ensure that a number of each machine type is within its utilisation capacity; otherwise its number will increase. Constraint 18 is to ensure that the sum of machining times of operations in duplicated machines of various parts in a cell is less than its capacity. Equations 19, 20, and 21 state the conditions to assign values for $U_{ijk,}$ $V_{ijk,}$ and IN_{ijk} as 0 or 1. Equations 22 and 23 are the condition to assign DN_{ijk} and SC_{ijk} as 1. Equations 24, 25, 26, and 27 are the conditions to assign M_{ijk}, $Z_{ijk,}$ $O_{ijk,}$ and W_{ijk} as 0 or 1 with preconditions DNijk, SCijk as 0 or 1.

Input data (*Case study 2: 10 machines X 8 components)*
The block diagonal form in Table 2 is converted into $X_{ik,}$ Y_{jk} as an input data set, along with the other data sets from Table 1. The incidence matrix of size M x C is the primary data input, given as Inc [M][C](refer to Table 1). The purchase price, machine duplication budget, and the capacity of each machine type are given as A[M], B[M], and C[M] respectively. The inter-cell moving cost, intra-cell moving cost, sub-contract price, past, present, and future demands, and production volume of each part type are given as I[C], IA[C], S[C], D[C], D1[C], and V[C] respectively.

Xik = [[1, 0, 1, 0, 0, 0, 1, 0, 1, 0],
[0, 1, 0, 0, 0, 1, 0, 1, 0, 0],
[0, 0, 0, 1, 1, 0, 0, 0, 0, 1]];

Yjk = [[1, 0, 0, 0, 0, 1, 0, 1],
[0, 0, 1, 0, 0, 0, 1, 0],
[0, 1, 0, 1, 1, 0, 0, 0]];

Step 3: If an exceptional part is assigned to two or more exceptional machines, then either all of these machines or none are duplicated in the cell to which the part was originally assigned.

Step 4: The constraint for duplication budget is formulated using the purchase price of those machines related to each bottleneck part.

4.2 Budgetary constraint

$$\sum_{k}^{c} \sum_{i \epsilon \mathbf{DM}}^{M} \sum_{j \epsilon \mathbf{SP}}^{C} \text{Mijk} \ \text{x Qi x} \ \text{Ai} \leq \text{Bik} \tag{30}$$

In the analysis of the exceptional elements, sometimes sub-contracting the bottleneck parts will be dealt with alone to check whether or not the bottleneck machines are to be considered for duplication. The optimisation model is used to solve all the case studies in Ilog Cplex 12.2. An engine log for case study 2 during execution is a minimisation problem with 448 variables and 635 constraints.

Table 4: Computational results: LPP OPL Cplex model solutions

Case study	₹ Z	Z_{ijk} units	M_{ijk} units	O_{ijk} units	W_{ijk} units	EE
Case study 1 7M X 8P	10,400	Z_{472}= 8 Z_{232}= 8	Mijk=0	O_{472}=80 O_{232}=75	Σ=70 (W_{ij1}= 16; W_{ij2}= 6)	2 (DM-0, SP-2)
Case study 2 10M X 8P	50,756	Z_{681}=9 Z_{732}=8 Z_{153}=8 Z_{953}=8	Mijk=0	O_{681}=250 O_{732}=280 O_{153}=300 O_{953}=300	Σ=72 (W_{ij1}=12; W_{ij2}= 6; W_{ij3}=8)	4 (DM-0, SP-4)
Case study 3 9M X 10P	1,83,020	Z_{411}=10, Z_{811}=10 Z_{811}=10, Z_{7103}=15	M_{821}=1 M_{242}, M_{262}=1 M_{773}=1	O_{411}=320 O_{811}=320 O_{7103}=280	Σ=300 (W_{321}= 4; W_{542}= 12, W_{673}=10)	7 (DM-4, SP-3)
Case study 4 10M X 12P	66,841	Z_{451}=5 Z_{851}=4 Z_{891}=5	M_{141}=1,	O_{451}=380 O_{851}=380 O_{891}=400	Σ=60 (W_{ij1}= 19; W_{ij2}=26, Wij3=15)	4 (DM-1, SP-3)
Case study 5 10M X 20P	2,320	Z_{ijk}=0	M_{ijk}=0	O_{ijk}=0	Σ=100 (W_{ij1}=15; W_{ij2}= 25; Wij3=30; Wij4=30)	0

Case study	₹ Z	Z_{ijk} units	M_{ijk} units	O_{ijk} units	W_{ijk} units	EE
Case study 6 18M X 24P	2,16,833	Z_{15171}=5, Z_{1591}=5 Z_{12171}=5, Z_{12931}=5 Z_{1262}=5, Z_{12222}=5, Z_{12132}=5 Z_{632}=5 Z_{1132}=5 Z_{13192}=5 Z_{8203}=5	M_{8241},M_{8231} M_{8101}=1 M_{15151}, M_{15171}=1 M_{542}, M_{5142}, M_{5112}, M_{5162}=1	O_{15171}=5, O_{1591}=5 O_{12171}=5, O_{12931}=5 O_{1262}=5, O_{12222}=5, O_{12132}=5 O_{632}=5 O_{1132}=5 O_{13192}=5 O_{8203}=5	Σ=115 (Wij_1=55; Wij_2=35; Wij3=25)	18 (DM-3, SP-15)

4.3 Discussion of results

In the solution of case study 3, out of seven exceptional elements, bottleneck machines 2, 7, and 8 are chosen to be duplicated and estimated for intra-movement costs concerning their cell; bottleneck components 2, 4, 6, and 7 are eliminated, and the remaining two (1 and 10) are sub-contracted. In most problems, the sub-contracting cost is low and moderate, so parts are sub-contracted alone to eliminate EE. The total of all intra-movements is given and, along with it, some movements are given — one in each cell. The duplication of machine 8 in case study 6 consecutively eliminates three exceptional elements; the duplication of machine 5 consecutively eliminates four exceptional elements; the duplication of machine 15 eliminates two exceptional elements. The number of duplicating machines in the respective cells depends upon the total machining time of the machine processing parts.

Table 5: Evaluation of results of optimisation model with best-known approaches comparing % savings with and without machine duplication and part sub-contracting

Case study	% Savings in cost by duplication and sub-contracting	Machine duplication cost M_{ijk} x A_{ik}	Intra-cell movement cost W_{ijk} x IA_j x BS_j	Parts sub-contracting cost O_{ijk} x S_j x D_j	Inter-cell movement cost Z_{ijk} x I_j x BS_j	Literature reference approaches used
Case study 1 7M X 8P	by proposed model ₹ 76.8%	0	4200	4850	1350	Mathematical model[23]
	by best-known methods $(U) (5MX7P) 76.65%	3644	306	-	856	
Case study 2 10M X 8P	by proposed model ₹ 67%	0	8100	40450	2206	Fuzzy multi-objective model [24]
	by best-known methods $(U) (8M X 12P) 63%	21405	11260	-	-	
Case study 3 9M X 10P	by proposed model ₹ 66%	160000	7090	12250	3680	Parametric model [11]
	by best-known methods $(U) 58%	286000	-	84420	8310	
Case study 4 10M X 12P	by proposed model ₹ 74%	35000	16201	12620	3020	Multi-functional model [12]
	by best-known methods $(C) 69%	1732550	201204	-	201032	
Case study 5 10M X 20P	by proposed model 94% ₹	0	2320	0	0	Multi-functional model [12]

Case study	% Savings in cost by duplication and sub-contracting	Machine duplication cost $M_{ijk} \times A_{ik}$	Intra-cell movement cost $W_{ijk} \times IA_j \times BS_j$	Parts sub-contracting cost $O_{ijk} \times S_j \times D_j$	Inter-cell movement cost $Z_{ijk} \times I_j \times BS_j$	Literature reference approaches used
Case study 5 10M X 20P (cont.)	by best-known methods $(U) (8M X 20P) 76%	3225230	303050	38737	82126	
Case study 6 18M X 24P	by proposed model 63% ₹	152000	31033	21200	12600	Mathematical model [23]
	by best known methods $(U) (7M X 14P) 72%	3330	-	-	440	

In evaluating the performance of the linear programming optimisation model against recently used approaches, the solutions to smaller problems are almost the same, but the proposed model yields good results for medium and large problems, particularly for duplication and sub-contracting.

Several inter- and intra-cellular movements; the number of parts to be contracted is given. In this proposed model, once the duplication of machines has been completed without sub-contracting parts, there is no need to calculate the inter-cell movements' costs and sub-contracting costs.

5 CELL LAYOUT DESIGN

The location of cells in a manufacturing industry considers the volume of the part supplied to each cell and the volume of the part allotted to each machine in the respective cell. The strategies followed in facility design are flexibility, optimum space use, and minimum capital investment. The part volume to be processed plays a vital role in locating the machinery within the cell and locating the cell within the shop floor, according to the determined preference order.

The aisle can be between 0.9m and 1.5m for small and medium-size layouts, and between 1.5m and 1.8m for larger layouts, depending on the available floor area. But for effective material handling and supervision, the minimum lengths of aisles — 0.9m and 1.5m — are preferred. In all of these case studies, the machines' width is assumed to be 1.2m for all the machines in smaller and medium-size cellular layouts.

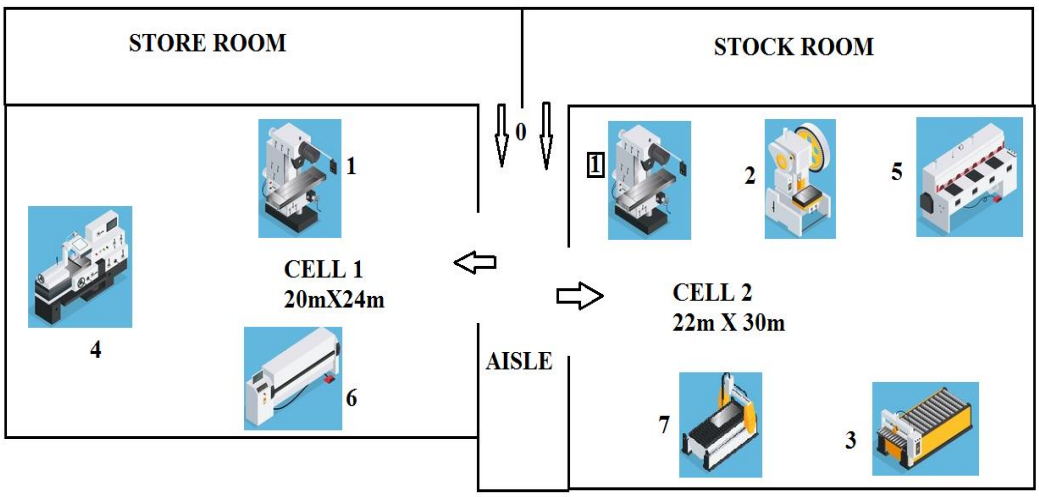

Figure 1: Cell layout design for case study 1 — 7 machines x 8 components

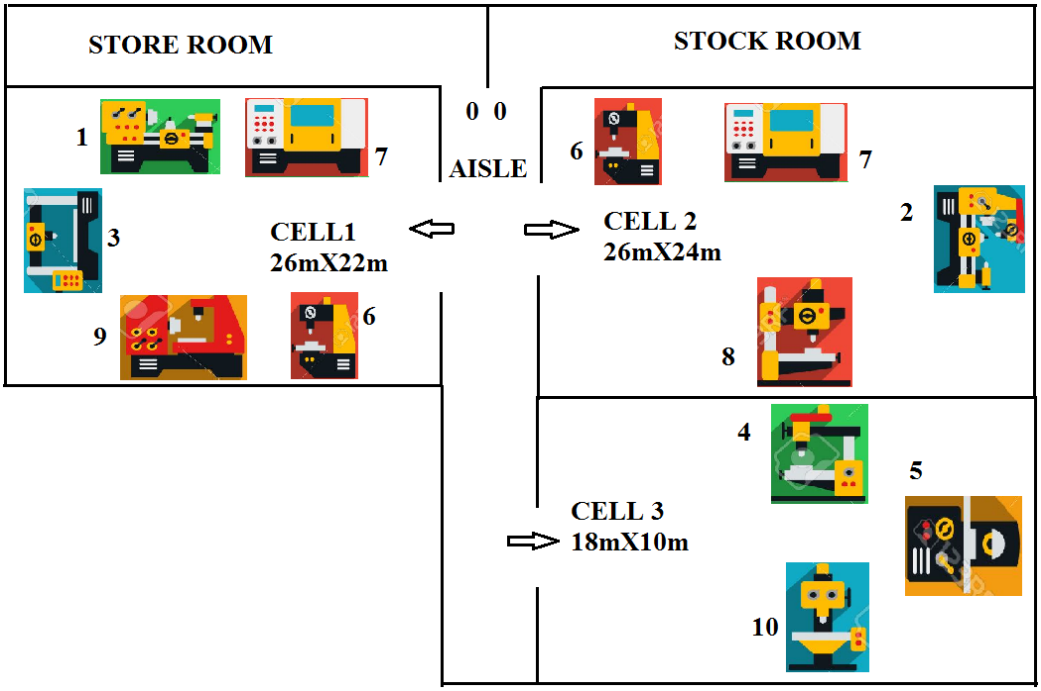

Figure 2: Cell layout design for case study 2 – 10 machines x 8 components

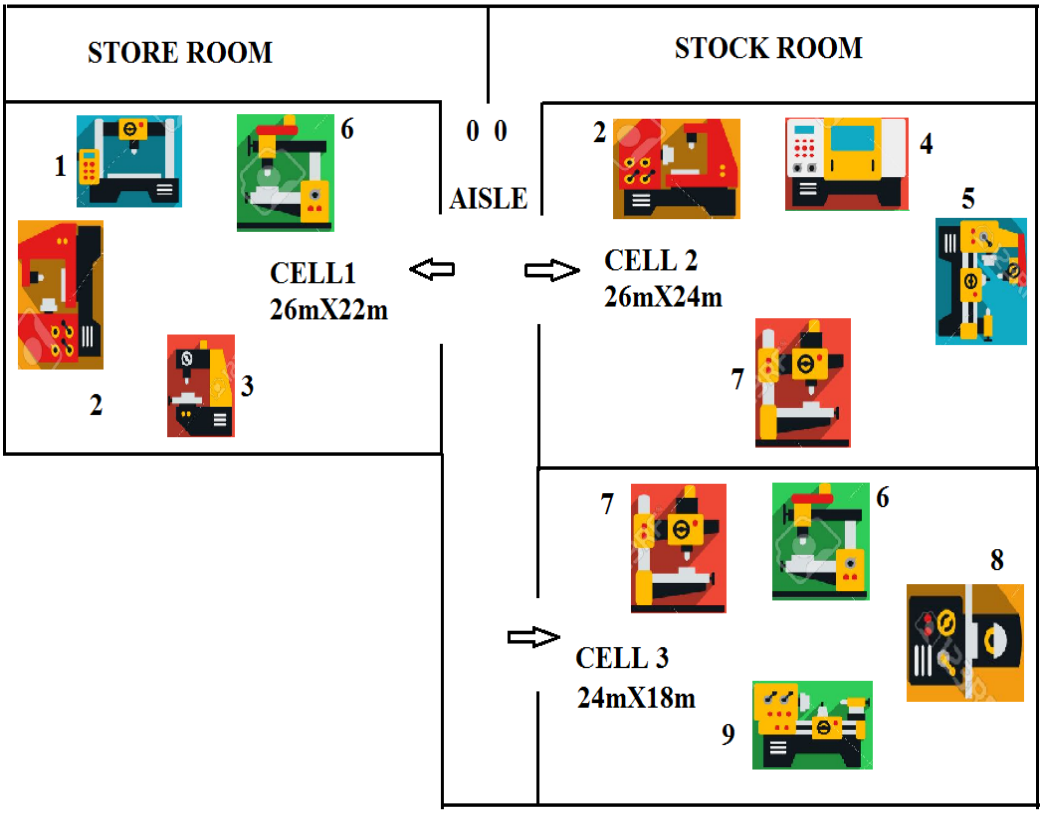

Figure 3: Cell layout design for case study 3 – 9 machines x 10 components

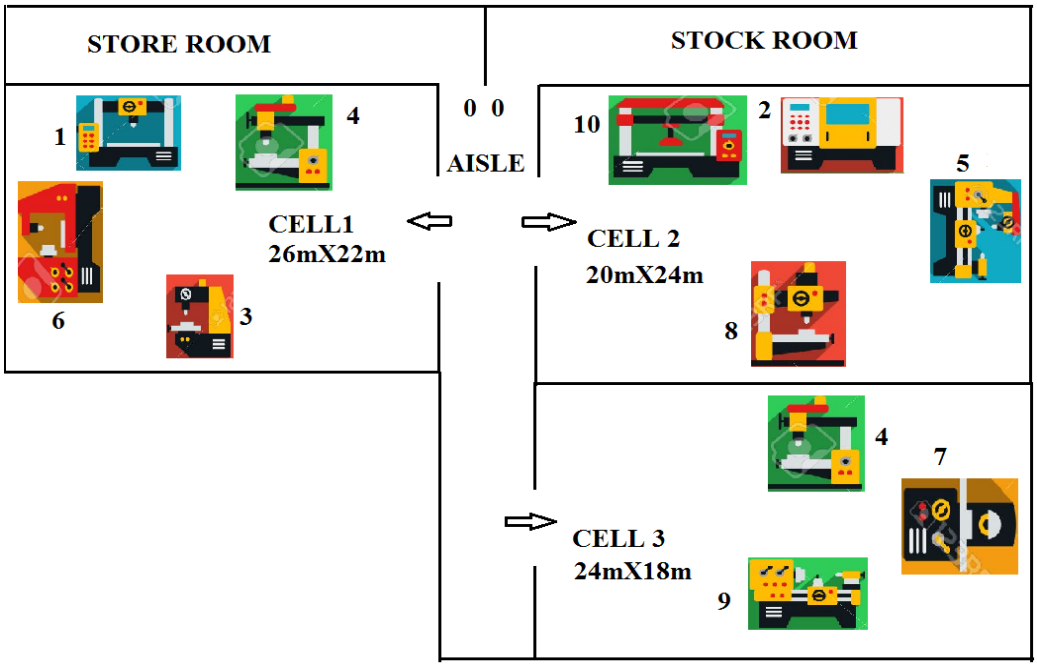

Figure 4: Cell layout design for case study 4 –10 machines x 12 components

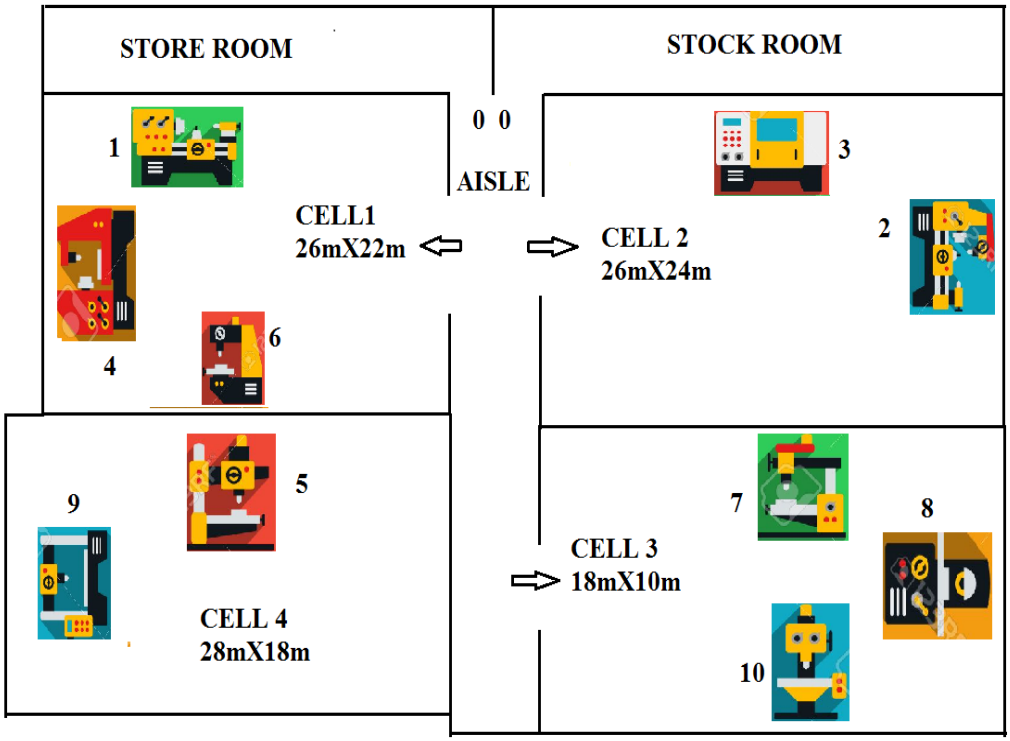

Figure 5: Cell layout design for case study 5 – 10 machines x 20 components

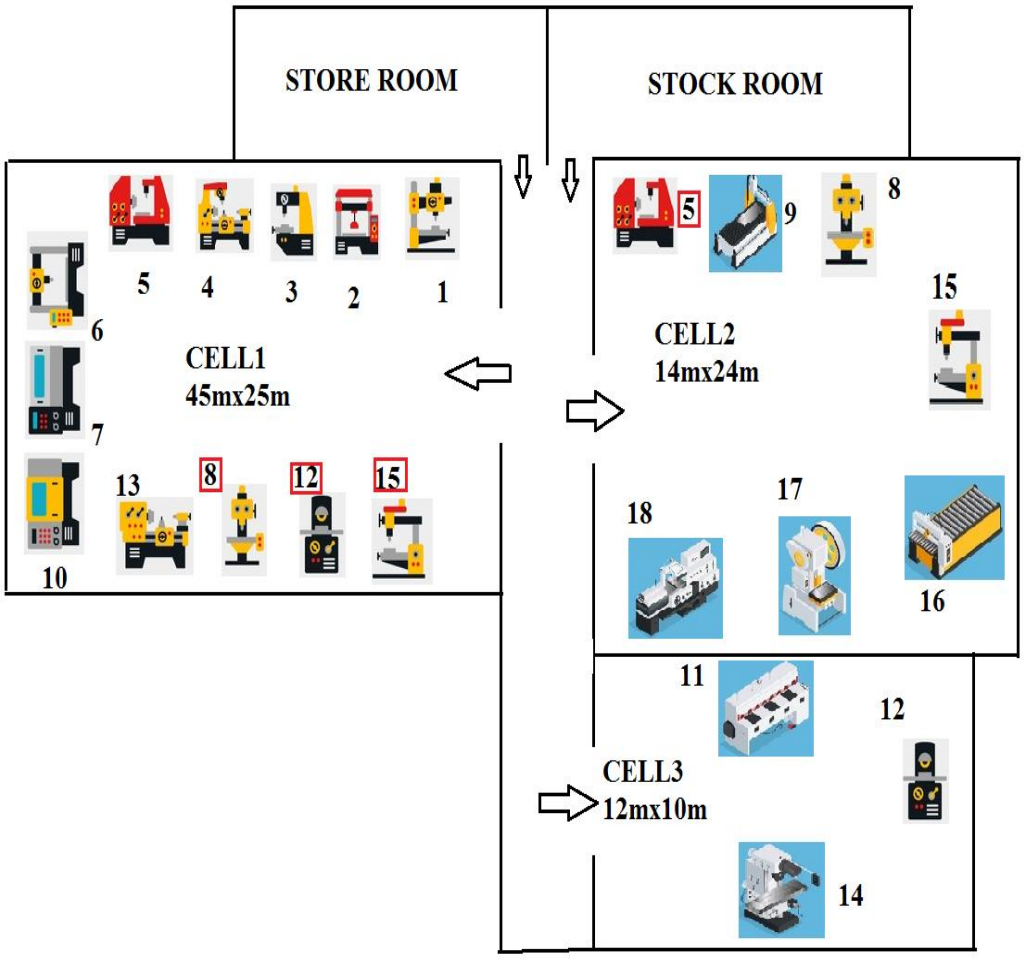

Figure 6: Cell layout design for case study 6 — 18 machines x 24 components

5.1 Number of machines in the cell (M_k)

The machines are arranged in a U-shaped layout to enable effective intra-movements of materials, tools, labour, and supervision over the entire cell. The machines are divided into three equal sets; the first two full sets of machines are arranged along the right side, lengthwise, and the remaining machines are arranged along the left side of the cell, starting from the point of entry.

5.2 Discussion of results

The cell layout design is prepared as a 2D shop floor plan, using a suitable scale to have easy measuring lengths. All of these cells and machines are located in relation to the storeroom and the stock room, with an adjacent entry as the starting point namely origin 0 x 0. The cell and machine locations can be measured as rectilinear from the origin for an easy plotting of the shop floor. The aisles of the cells, machines, and partitions are considered in line with the problem size. The outcomes obtained from this layout design are the floor area required by each cell and savings in the floor area compared with the traditional process layout, and the distances travelled by each job in and between cells and the savings in distances travelled compared with the process layout. The savings in floor area and movement length are not related to the case study size, but depend solely on the types of machinery provided and the part volumes handled.

6 CONCLUSIONS

The best clusters of machine cells and part families are achieved simultaneously, compared with recent mathematical approaches to using the floor area saving, to reduce human movements and manufacturing costs. The results of the optimisation mathematical model are the costs of EE, while the estimation of the

costs of duplication, sub-contracting, and inter- and intra-movements seriously considered the budgetary constraints of duplication and the economic trade-off of parts sub-contracting.

The performance evaluation proves the sustainability of the proposed PARI model, and that it is suitable for small and medium-size formation case studies, yielding greater grouping efficiency and working much better for large formation case studies with a lower percentage of exceptional elements than other mathematical approaches. In this proposed model, once the duplication of machines is done without sub-contracting the parts, there is no need to calculate the inter-cell movement's costs and the sub-contracting costs. Significance will be given in cases when there are both inter- and intra-cell movement costs if duplication or sub-contracting arises. Because of management's investment policy, increases in machine duplication will be restricted owing to budgetary constraints; and this will tend to increase the sub-contracting of parts.

In future, the proposed mathematical approach could be extended for scheduling as well as line balancing through heuristic clustering by considering simulation on the shop floors of warehouses and storerooms, as well as inventory for raw materials — all being a work-in-process.

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