A NEW MODEL FOR SINGLE FACILITY LOCATION BASED ON SERVICE LEVEL

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ABSTRACT

This paper studies a new continuous single facility location problem. In this problem the locations of the customers (or the previously located facilities) vary, randomly, and the objective is to locate a new facility so as to maximise the mean service level considered to be the mean number of the customers whose distances from the new facility are smaller than a predefined desirable distance. The problem is formulated, and then an approximate solution method is presented to solve the problem. Also, we apply our results to locate the capital of Iran as a case study.

OPSOMMING

Die navorsing behandel 'n enkele vestigingsvraagstuk. Die probleem vereis bepaling van die maksimum waarde van gemiddelde dienspeil aan verbruikers teen 'n agtergrond van toevalligverdeelde afsetpunte. 'n Benaderde oplossingsmetode word voorgehou. By wyse van illustrasie word die metode gebruik om die gewensde geografiese posisie van die Iranese hoofstad te bepaal.

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1. INTRODUCTION

Facility location problems are about optimally locating new facilities to serve previously located customers or old facilities. We refer to the books by Daskin [2], Drezner [3], or Drezner & Hamacher [4] for an introduction to deterministic facility location theory, and to Louveaux [5] and Snyder [6] for a comprehensive survey of issues and models regarding stochastic and uncertain facility location.

In this article we study a new continuous single facility location problem called the *Facility Location based on Service Level* (FLSL) problem. We explain it below.

- **Parameters**:

m: The number of customers or previously located facilities.
(X_i, Y_i): The location of the *i*th customer (or the *i*th previously located facility) where $X_i, Y_i,\ i = 1, \ldots, m,$ are arbitrary random variables.
dd: A given parameter representing the desirable distance for all the customers.

- **Decision variables:**

(x, y): The location of the new single facility.

- **Objective:**

Maximisation of the mean number of customers whose distances from the new facility are smaller than the desirable distance dd.

Generally *service level* is a measure of the performance of a system, and is defined as the number of well-served customers out of the total number of served customers; the ratio between the services that are desirable for customers and the total services; or the percentage of fulfilled services out of all services ordered in the system. In fact, in our problem we can define the service level of the new facility as the number of customers whose distances from the new facility are desirable, and so the objective of our problem can be interpreted as the maximisation of the mean service level of the new facility. The desirable distances of customers may be different, but to simplify the problem without loss of generality we assume that all the desirable distances are the same. We also assume that all the distances are Euclidean, although other types of distances can be considered.

To compare our new model with the old single facility location models, we present a brief review of the most important traditional approaches. Traditionally, under assumptions similar to our problem, when the locations of customers are precise, i.e. $\Pr\{(X_i, Y_i) = (x_i, y_i)\} = 1,\ i = 1, \ldots, m$, locating the new facility is considered by means of several classical problems. Two of the most important ones are the 1-Median and 1-Center problems. The 1-Median (or Weber or MinSum) problem can be written as:

$$\min_{(x,y)} \sum_{i=1}^{m} w_i d((x,y),(x_i,y_i)) \qquad \text{1-Median} \qquad (1.1)$$

where $d : \Re^2 \times \Re^2 \to \Re$ is a meter on $\Re^2$ and is used to measure the distances between the new facility and old facility locations. Also, $w_i, i = 1, \ldots, m$, are the weights considered for the old facilities to encounter their relative importance with respect to the new facility. Two common choices for the distance meter d are the rectilinear distance d_{RL} (or Manhattan distance or taxicab metric) and the Euclidean distance d_E, which are defined respectively as:

$$d_{RL}((x,y),(x_0,y_0)) = \|(x-x_0, y-y_0)\|_1$$
$$d_E((x,y),(x_0,y_0)) = \|(x-x_0, y-y_0)\|_2 \,.$$

where $\|.\|_p : \Re^2 \to \Re$ is the l_p norm on $\Re^2$ and defined as $\|(x,y)\|_p = \left(|x|^p + |y|^p\right)^{\frac{1}{p}}$ for $p \geq 1$.

The 1-Center (or MinMax) problem can be formulated as:

$$\min_{(x,y)} \max_{i=1,\ldots m} \{w_i d((x,y),(x_i,y_i))\}\,. \qquad \text{1-Center} \tag{1.2}$$

Both the 1-Median and 1-Center problems can be straightforwardly extended for cases that the locations of the customers are random as:

$$\min_{(x,y)} \mathrm{E}\left[\sum_{i=1}^{m} w_i d((x,y),(X_i,Y_i))\right] \text{ Stochastic 1-Median} \tag{1.3}$$

$$\min_{(x,y)} \mathrm{E}\left[\max_{i=1,\ldots m} \{w_i d((x,y),(X_i,Y_i))\}\right] \text{ Stochastic 1-Center} \tag{1.4}$$

For more references and details concerning the above models, we refer to Daskin [2], Drezner & Hamacher [4], and Snyder [6].

As we see in models (1.1) to (1.4), their goals are to locate the new facility such that a general performance measure – e.g. the sum or maximum of transportation costs or total weighted distances – is optimised. However, they do not directly attempt to improve some service level based on customer desirability. Consequently we are motivated to introduce and study the FLSL problem in this paper. This study could be very beneficial in the field of industrial engineering and management because in the past two decades the design of systems that consider customer desirability have gained more and more attention from companies and supply chains.

The rest of this paper is organised as follows. Section 2 studies the problem and presents an exact solution method. Section 3 gives an approximate method. In Section 4 the case study is presented, and several experiments are conducted to compare the new model with traditional models. Section 5 concludes the paper.

2. FORMULATION AND EXACT SOLUTION METHOD

The FLSL problem can be formulated as the following optimization problem:

$$\max_{(x,y)} \mathrm{E}\{N_{(x,y)}\} \text{ FLSL} \tag{2.1}$$

where $N_{(x,y)}$ denotes the number of satisfied constraints in the set of uncertain constraints

$$S_{(x,y)} = \left\{ \|(X_i - x, Y_i - y)\|_2 \leq dd,\ i = 1,2,\ldots,m \right\}.$$

It should be noted that this problem is related to the yield maximisation problem (YM problem), in which we have several *hard constraints,* and we try to maximise the probability that all constraints are satisfied (see Ahmadi-Javid & Seifi [1] for more details);while in the FLSL problem we have several *soft constraints* and we try to maximise the mean number of satisfied constraints.

To solve the optimisation problem (2.1), first the expectation $\mathrm{E}\{N_{(x,y)}\}$ should be evaluated practically. The following proposition shows how this expectation can be computed.

Proposition 2.1.

$$\max_{(x,y)} \mathrm{E}\{N_{(x,y)}\} = \sum_{i=1}^{m} \Pr\{\|(X_i - x, Y_i - y)\|_2 \le dd\}.$$

Proof. The random variable $N_{(x,y)}$ can be written as the sum of the random variables $I_i, i = 1,\ldots,m$,

$$N_{(x,y)} = \sum_{i=1}^{m} I_i$$

where $I_i, i = 1,\ldots,m$, are defined as

$$I_i = \begin{cases} 1 & \|(X_i - x, Y_i - y)\|_2 \le dd \\ 0 & \|(X_i - x, Y_i - y)\|_2 > dd \end{cases}, i = 1,\ldots,m.$$

Furthermore, we have:

$$\mathrm{E}(I_i) = \Pr\{\|(X_i - x, Y_i - y)\|_2 \le dd\}, \; i = 1,\ldots,m.$$

Thus, the proof is finished.

Model (2.1) is a nonconvex optimisation problem with only two decision variables. When the joint support of the random variables $X_i, Y_i, \; i = 1,\ldots,m$, is bounded, the global optimum can be obtained easily by the direct search method with a given accuracy.

The next proposition presents a special case of the FLSL problem when the locations of the customers are deterministic. In this case the FLSL problem can be formulated as a mixed integer convex program.

Proposition 2.2. When $\Pr\{(X_i, Y_i) = (x_i, y_i)\} = 1, \; i = 1,\ldots,m$, the FLSL problem can be written as the following mixed integer convex program:

$$\begin{aligned} &\max \; \sum_{i=1}^{m} z_i \\ &\|(x_i - x, y_i - y)\|_2 \le dd + M(1 - z_i) \quad i = 1,\ldots,m \\ &x, y \in \Re \\ &z_i : 0,1 \quad i = 1,\ldots,m. \end{aligned}$$

Proof. The proof follows from Proposition 2.1.

3. APPROXIMATE SOLUTION METHOD

The optimisation problem given in (2.1) is a nonconvex program, and cannot be solved globally unless we use the direct search method. Also, the evaluation or estimation of $\mathrm{E}\{N_{(x,y)}\}$ is very time-consuming. Therefore, in this section we present an efficient approximate method. To develop this method, first we present an approximation for $\mathrm{E}\{N_{(x,y)}\}$, and then we use it to approximate Model (2.1) by a mixed integer convex program. The approximation for $\mathrm{E}\{N_{(x,y)}\}$ is:

$$\mathrm{E}\{N_{(x,y)}\} \approx n_{(x,y)}$$

where $n_{(x,y)}$ is the number of satisfied constraints in the set

$$S'_{(x,y)} = \left\{ \mathrm{E}\left(dd^2 - \|(X_i - x, Y_i - y)\|_2^2 \right) \geq 0 : i = 1,2,\ldots,m \right\}.$$

Note that the set $S'_{(x,y)}$ is obtained from

$$S_{(x,y)} = \left\{ sl^2 - \|(X_i - x, Y_i - y)\|_2^2 \geq 0 : i = 1,2,\ldots,m \right\}$$

by replacing the terms $dd^2 - \|(X_i - x, Y_i - y)\|_2^2$, $i = 1,2,\ldots,m$, with their expectations.

Now Model (2.1) can be approximated by the program:

$$\begin{aligned} &\max \sum_{i=1}^{m} z_i \\ &\mathrm{E}\left(dd^2 - \|(X_i - x, Y_i - y)\|_2^2 \right) \geq -M(1 - z_i) \quad i = 1,\ldots,m \\ &z_i : 0,1 \quad i = 1,\ldots,m \end{aligned} \tag{3.1}$$

where M is a big number. The next proposition studies this problem.

Proposition 3.1. Model (3.1) can be written as the following mixed integer convex program:

$$\begin{aligned} &\max \sum_{i=1}^{m} z_i \\ &dd^2 + M(1 - z_i) \geq \|(\mathrm{E}(X_i) - x, \mathrm{E}(Y_i) - y)\|_2^2 + \mathrm{var}(X_i) + \mathrm{var}(Y_i) \quad i = 1,\ldots,m \\ &z_i : 0,1 \quad i = 1,\ldots,m. \end{aligned} \tag{3.2}$$

Also, the above formulation is equivalent to the formulation given in Proposition 2.2 when the locations of customers are deterministic.

Proof. For the *i*th constraints of model (3.1) we have:

$$\mathrm{E}\left\{ -\|(X_i - x, Y_i - y)\|_2^2 + dd^2 \right\} = -\mathrm{E}\left\{ (X_i - x)^2 + (Y_i - y)^2 \right\} + dd^2 .$$

By expectation and reformulation,

$$\mathrm{E}\left\{ -\|(X_i - x, Y_i - y)\|_2^2 + dd^2 \right\} = -\|(\mathrm{E}(X_i) - x, \mathrm{E}(Y_i) - y)\|_2^2 - \mathrm{var}(X_i) - \mathrm{var}(Y_i) + dd^2$$

Thus, the proof is completed.

4. CASE STUDY AND COMPUTATIONAL RESULTS

This section presents a case study for the FLSL problem. Here the location of the capital of Iran is found based on service level. The service level is considered as the number of persons who can travel to the capital by covering a desirable distance. Indeed, the location of a person who decides to travel to the capital (from the perspective of the designer) is not completely known, and also may be dynamically changed, so the locations of persons are considered uncertain. To model this uncertainty, we assume that location of each person is uniformly distributed in a rectangular area. The people of Iran are classified into 35 classes by considering their location resemblance. Figure 1 depicts these 35 rectangles.

The objective is to find the location of the capital such that the mean number of people, whose distances from the capital are less than a desirable distance, is maximised. Using model (2.1) the optimal location of the capital is obtained for $0 < dd \leq 50$ by a direct search to an accuracy of ± 0.25 (each unit of the desirable distance is equal to 25 km). The mean number of satisfied persons in relation to each point in the direct search method is approximated by a Monte Carlo simulation of 10,000 samples – a number that is found to be sufficient for this problem, as the results were not sensitive beyond 10,000. Also, the approximate locations of the capital are found by model (3.2) for $0 < dd \le 50$. The model of the approximate method is solved by Lingo 8, and the direct search and simulation procedures are coded in Visual Basic 6 and implemented in Excel spread sheets. For each case the solution time for the exact method is about 3 hours, and the solution time by Lingo 8 is less than 5 seconds (CPU: Pentium 4 with 2.8 GB processor, RAM: 512 MB).

The results of the exact and approximate methods for several values of desirable distance are given in Table 1. From this table it can be seen that the approximate method can give suitable results.

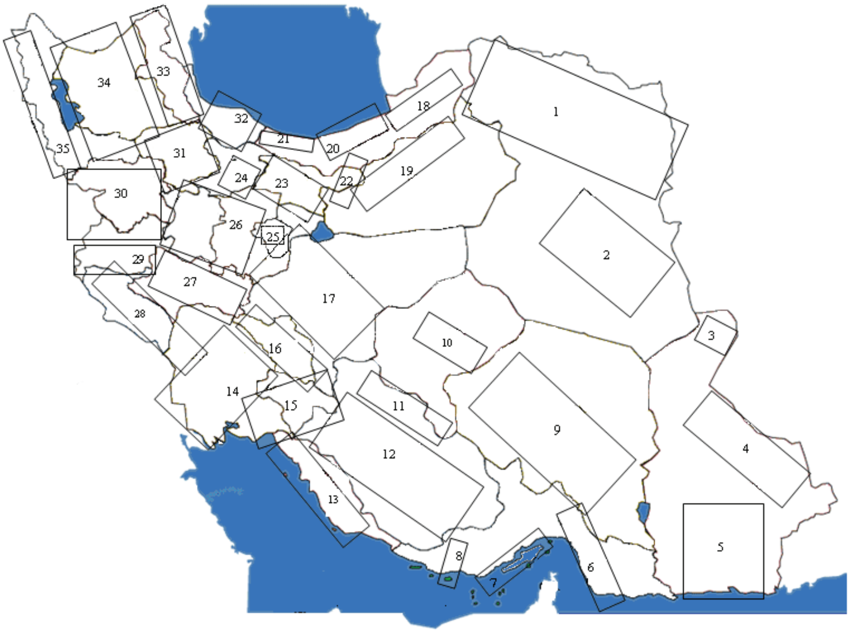

Figure 1: The plot of the rectangles of the 35 possible locations for the people of Iran

dd	Exact method	Approximate method
0^+	0.00	0.00
2.5	0.10	0.10
5	0.20	0.16
9	0.33	0.32
15	0.52	0.51
20	0.65	0.64
25	0.74	0.68
30	0.87	0.85
35	0.94	0.94
40	0.98	0.96
45	1.00	0.98
50	1.00	1.00

Table 1: The mean service levels obtained by the exact and approximate methods

In Table 2 we report the improvement percentages gained by our model over eight traditional models. The eight models are models (1.1) to (1.4), considering both rectilinear and Euclidean distances. For models (1.1) and (1.2), the locations of the people are considered to be their expected locations. To solve the deterministic models we use Lingo 8, and stochastic models are solved by the direct search method to an accuracy of ± 0.25.

From Table 2 it can be seen that using the new model dramatically improves the mean service levels, especially for medium values of desirable distances which are more important in the real world. Moreover, generally the results obtained by the models based on the 1-Median problem (i.e. columns 5 to 8) are vastly better for $0 < dd \leq 30$, and for large values of desirable distance they are similar to, or a little worse than, the results obtained by the models based on the 1-Center problem. This leads us to the fact that the models based on the 1-Median problem are more reliable and conservative than the ones based on the 1-Center problem with respect to service level. Also, we observe that the deterministic models (see columns 1, 2, 5 and 6) give better results for the 1-Medium problem when $0 < dd \leq 10$ and for the 1-Center problem when $10 \leq dd \leq 40$. In addition, for the 1-Medium problem when $10 < dd$, the results obtained by the stochastic models are a little better than the results obtained by the deterministic versions of those models. This goes somewhat against our initial anticipation that the stochastic models generally perform better in solving our problem, due to the randomness of the customer locations. Also, as we expected, because the desirable distance is Euclidean, we see that the models based on Euclidean distance mostly perform better.

5. CONCLUSION

In single facility location problems, the objective is to locate a new service facility to serve the customers or old facilities optimally. The important issue in the classical single facility location problems is to locate the new facility such that a general performance measure – e.g. the sum or maximum of transportation costs or total transportation distances – is optimised. However, these problems do not directly attempt to improve the service level based on customer desirability. This motivates us to introduce and study a new continuous single facility location problem based on service level. In this problem we assume that the locations of customers vary randomly, and the objective is to maximise the mean of the service level, defined as the mean number of customers whose distances from the new facility are smaller than a desirable distance.

The problem is modeled and solved both exactly and approximately. Then the model is implemented to determine the location of the capital of Iran based on service level. The desirable distance is considered to be that which is suitable for a person who intends to travel to the capital. Since the location of a person wanting to travel to the capital is uncertain and unpredictable, it is assumed that the location of each person is uniformly distributed in a rectangular area. The people of Iran are classified into 35 classes by considering their location resemblance. Then the locations of the capital are found for several values of desirable distance by the exact and approximate methods. The results show that the approximate method can perform promisingly.

In the next section we conduct several experiments to compare our model with the eight classical models, corresponding to the deterministic and stochastic 1-Median and 1-Center problems, by considering the rectilinear and Euclidean distances. It is shown that using the new model dramatically improves the mean service level, especially for medium desirable distance values, which are more important in practical situations. Moreover, generally the results obtained by the models based on the 1-Median problem are vastly better for small and medium desirable distance values, while for large values of desirable distance they are similar to or a little worse than the results obtained by the models based on the 1-Center problem. This indicates that, from the viewpoint of the service level defined here, if we use the models based on the 1-Median problem, the results are more reliable and conservative than those that are obtained based on the 1-Center problem. Also, we observe that the deterministic models give better results for the 1-Median problem when the desirable distance values are small. In addition, when the desirable distances are medium or large, the results obtained by the stochastic models are only a little better than the results obtained by the deterministic versions of the models. Similarly, the deterministic models based on the 1-Center problem give better results for a wide range of desirable distance values. This is somewhat against our expectation that the stochastic models generally perform better for our problem due to the randomness of the customer locations.

	Model							
	1-Median				1-Center			
	Deterministic		Stochastic		Deterministic		Stochastic	
dd	RL	E	RL	E	RL	E	RL	E
0^+	0%	0%	0%	0%	0%	0%	0%	0%
2.5	0%	53%	10%	64%	32704%	25758%	1772%	1547%
5	3%	5%	7%	5%	2912%	2204%	874%	790%
10	14%	13%	15%	13%	116%	109%	424%	393%
15	17%	12%	15%	12%	37%	37%	136%	133%
20	13%	7%	12%	7%	25%	25%	27%	27%
25	2%	1%	1%	1%	10%	10%	11%	12%
30	4%	4%	3%	4%	2%	2%	6%	6%
35	4%	3%	3%	3%	1%	1%	4%	4%
40	3%	3%	3%	3%	0%	0%	2%	2%
45	3%	3%	2%	3%	1%	1%	0%	0%
50	2%	2%	2%	2%	1%	1%	0%	0%

Table 2: The improvement percentages obtained by the proposed model over the eight traditional models

From our study, we find that directly optimising the service level yields significant improvement when our main objective is to improve the service level of the system based on some specific desirable distance for customers. This observation is very important for designing customer-based systems in competitive business environments. Moreover, when our main objective is not the optimisation of the service level, it is very helpful for managers and industrial designers to evaluate the selected approach, or to compare possible approaches from the viewpoint of service level.

6. REFERENCES

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