PROJECT ACTIVITY ANALYSIS WITHOUT THE NETWORK MODEL

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ABSTRACT

This paper presents a new procedure for analysing and managing activity sequences in projects. The new procedure determines critical activities, critical path, start times, free floats, crash limits, and other useful information without the use of the network model. Even though network models have been successfully used in project management so far, there are weaknesses associated with the use. A network is not easy to generate, and dummies that are usually associated with it make the network diagram complex – and dummy activities have no meaning in the original project management problem. The network model for projects can be avoided while still obtaining all the useful information that is required for project management. What are required are the activities, their accurate durations, and their predecessors.

OPSOMMING

Die navorsing beskryf 'n nuwerwetse metode vir die ontleding en bestuur van die sekwensiële aktiwiteite van projekte. Die voorgestelde metode bepaal kritiese aktiwiteite, die kritieke pad, aanvangstye, speling, verhasing, en ander groothede sonder die gebruik van 'n netwerkmodel. Die metode funksioneer bevredigend in die praktyk, en omseil die administratiewe rompslomp van die tradisionele netwerkmodelle.

1. INTRODUCTION

Networks have been used successfully in project management since the 1950s. Networks graphically show the total amount of time needed to complete a project, the sequence in which the tasks must be carried out, the critical tasks that need close attention, and which tasks can be carried out simultaneously [2] [4] [6] [8]. A project manager can also shorten the project duration by adding more resources to certain tasks in an attempt to get them done faster. The network diagram has proved to be a useful tool for scheduling activities in a project [1] [7]. When unexpected circumstances cause slight changes in durations – for example, a worker strike, resources supply problems, or unpredictable weather – such problems require the rescheduling of activities and rapid computation. Changing networks are called protean networks [3], and for very large projects a slight delay in decision-making can be costly. The network diagram can be avoided while still obtaining the same scheduling decisions. This paper presents a novel procedure for analysing and managing activity sequences in projects. The procedure determines critical activities, the critical path, start times, free floats, crash limits, and other useful information without using a network model. Even though the network model has been used successfully so far in project management, there are weaknesses associated with it. A network is not easy to generate, and dummies that are usually associated with it make the network diagram complex – and they have no meaning in the original project management problem. One can avoid the network model for projects and still get all the useful information that is required for project management. What are required are the activities, their accurate duration estimates, and their predecessors. The proposed procedure changes are only incorporated into the affected activities; unaffected activities are not considered. As a result of this, technique calculations are rapidly carried out, resulting in timeous decisions.

Consider a given activity A^i , its ri predecessors, and an accurate duration estimate, as shown in Table 1 below.

Activity	Predecessor	Accurate duration estimate
A^1	$A_1^1, A_2^1, \ldots, A_{r1}^1$	d_1
...	...	...
A^i	$A_1^i, A_2^i, \ldots, A_{ri}^i$	d_i
...	...	...
A^m	$A_1^m, A_2^m, \ldots, A_{r1}^m$	d_m
...	...	...
A^n	$A_1^n, A_2^n, \ldots, A_{rn}^n$	d_n

Table 1

where $i = 1,2,\ldots,m,\ldots,n$

The data in Table 1 can be used to determine the critical activities, critical path, start and end times, free floats, crash limits, and other useful information that is required for project management without the use of a network diagram.

2. GENERATING THE LATEST END TIME (T_{LE}^i) FOR ACTIVITY A^i

The latest end time (T_{LE}^i) for activity (A^i) is given by

$$T_{LE}^i = \max\left[T_{LE1}^i, T_{LE2}^i, \ldots, T_{LEri}^i\right] + d_i \tag{1}$$

where T_{LEj}^{i} is the end time for j^{th} of the *ri* predecessors. In this case $j = 1,2,...,ri$

3. UPDATING LATEST END TIMES

Immediately after obtaining T_{LE}^{i} all the predecessors must have the same end time value of , where $T_{LE}^{i\max}$ is given by

$$T_{LE}^{i\max} = \max\left[T_{LE1}^{i}, T_{LE2}^{i},..., T_{LEri}^{i}\right] \quad (2)$$

The process is called 'updating', and changes the latest end times of all predecessors to T_{LE}^{i}.

4. CRITICAL ACTIVITIES

The activity giving the largest end time $(T_{LE}^{i\max})$ is the critical activity. The set of critical activities in chronological order is the critical path. This path is conveniently traced from the bottom of the table, going backwards. The latest start time of an activity is the latest end time of its predecessor.

5. GENERATING THE EARLIEST END TIME T_{EE}^{i} FOR ACTIVITY A^{i}

The earliest end time T_{EE}^{i} for activity A^{i} is generated from

$$T_{EE}^{i} = \min\left[(T_{EE1}^{si} - d_{s1}), (T_{EE2}^{si} - d_{s2}),..., (T_{EEk}^{si} - d_{sk})\right] \quad (3)$$

where T_{EEj}^{si} and d_{sj} are the earliest end time and duration of the successor activity A^{si} respectively. $j = 1,2,...,k.$ The updated end times and critical activities are shown in Table 2.

6. CRASHING ACTIVITY DURATIONS

'Crashing' refers to a technique used in project management for the purpose of decreasing the total project duration. Crashing is done after a careful and thorough analysis of all activities, their sequences and importance, so as to obtain the most convenient duration at the least additional cost. There are several approaches to crashing a project schedule. One of these is the minimum incoming weight label (*MIWL*) method proposed by Munapo et al. [5]. This method selects only those activities that are affected by crashing, and uses them to calculate the crash limit. It is efficient, but it does not make start and end times readily available, and it is also directly based on the project network diagram. The other and most common approach is the use of the smallest free float selected from *all* the noncritical activities as the crash limit. A serious drawback of this approach is that it uses all the noncritical activities to determine the crash limit. Some of these noncritical activities are not affected by the crashing, and as a result may give very small values. The technique proposed in this paper is efficient, it selects only those activities that are affected by crashing, and it uses them to calculate the crash limit. Both activity start and end times are also made readily available. Suppose activity A^{i} is selected for crashing, and activity A^{m} is one of the terminal critical activities. The normal project duration (T_{nor}^{P}) is given by

Activity	Predecessor	Accurate duration estimate
$A^1\begin{bmatrix}T_{LE}^1\\T_{EE}^1\end{bmatrix}$	$A_1^1\begin{bmatrix}T_{LE1}^1\\T_{EE1}^1\end{bmatrix}, A_2^1\begin{bmatrix}T_{LE2}^1\\T_{EE2}^1\end{bmatrix},\ldots, A_{r1}^1\begin{bmatrix}T_{LEr1}^1\\T_{EEr1}^1\end{bmatrix}$	d_1
...	...	...
$A^i\begin{bmatrix}T_{LE}^i\\T_{EE}^i\end{bmatrix}$	$A_1^i\begin{bmatrix}T_{LE1}^i\\T_{EE1}^i\end{bmatrix}, A_2^i\begin{bmatrix}T_{LE2}^i\\T_{EE2}^i\end{bmatrix},\ldots, A_{r1}^i\begin{bmatrix}T_{LEri}^i\\T_{EEri}^i\end{bmatrix}$	d_i
...	...	...
$A^m\begin{bmatrix}T_{LE}^m\\T_{EE}^m\end{bmatrix}$	$A_1^m\begin{bmatrix}T_{LE1}^m\\T_{EE1}^m\end{bmatrix}, A_2^m\begin{bmatrix}T_{LE2}^m\\T_{EE2}^m\end{bmatrix},\ldots, A_{r1}^m\begin{bmatrix}T_{LErm}^m\\T_{EErm}^m\end{bmatrix}$	d_m
...	...	...
$A^n\begin{bmatrix}T_{LE}^n\\T_{EE}^n\end{bmatrix}$	$A_1^n\begin{bmatrix}T_{LE1}^n\\T_{EE1}^n\end{bmatrix}, A_2^n\begin{bmatrix}T_{LE2}^n\\T_{EE2}^n\end{bmatrix},\ldots, A_{rn}^n\begin{bmatrix}T_{LErn}^n\\T_{EErn}^n\end{bmatrix}$	d_n

Suppose activity A^i is critical: then it is denoted by an asterisk, as follows:

$*A^i\begin{bmatrix}T_{LE}^i\\T_{EE}^i\end{bmatrix}$	$A_1^i\begin{bmatrix}T_{LE1}^i\\T_{EE1}^i\end{bmatrix}, A_2^i\begin{bmatrix}T_{LE2}^i\\T_{EE2}^i\end{bmatrix},\ldots, A_{r1}^i\begin{bmatrix}T_{LEri}^i\\T_{EEri}^i\end{bmatrix}$	d_i

Table 2

$$T_{nor}^{P} = T_{LE}^{m} \tag{4}$$

The earliest end times are updated as shown in Table 3.

Activity	Predecessor	Accurate duration estimate
$A^1\begin{bmatrix}T_{LE}^1\\ \ \end{bmatrix}$	$A_1^1\begin{bmatrix}T_{LE1}^1\\ \ \end{bmatrix}, A_2^1\begin{bmatrix}T_{LE2}^1\\ \ \end{bmatrix},\ldots, A_{r1}^1\begin{bmatrix}T_{LEr1}^1\\ \ \end{bmatrix}$	d_1
...	...	...
$*A^i\begin{bmatrix}T_{LE}^i\\ \ \end{bmatrix}$	$A_1^i\begin{bmatrix}T_{LE1}^i\\ \ \end{bmatrix}, A_2^i\begin{bmatrix}T_{LE2}^i\\ \ \end{bmatrix},\ldots, A_{r1}^i\begin{bmatrix}T_{LEri}^i\\ \ \end{bmatrix}$	d_i
...	...	...
$*A^m\begin{bmatrix}T_{LE}^m\\ \ \end{bmatrix}$	$A_1^m\begin{bmatrix}T_{LE1}^m\\ \ \end{bmatrix}, A_2^m\begin{bmatrix}T_{LE2}^m\\ \ \end{bmatrix},\ldots, A_{rm}^m\begin{bmatrix}T_{LErm}^m\\ \ \end{bmatrix}$	d_m
...	...	...
$A^n\begin{bmatrix}T_{LE}^n\\ \ \end{bmatrix}$	$A_1^n\begin{bmatrix}T_{LE1}^n\\ \ \end{bmatrix}, A_2^n\begin{bmatrix}T_{LE2}^n\\ \ \end{bmatrix},\ldots, A_{rn}^n\begin{bmatrix}T_{LErn}^n\\ \ \end{bmatrix}$	d_n

Table 3

Assume the crash limit to be cl. The duration d_i for the critical activity A^i is reduced by cl units, and the necessary recalculations made are shown in Table 4.

Activity	Predecessor	Accurate duration estimate
$A^1\left[T_{LE}^1\right]$	$A_1^1\left[T_{LE1}^1\right], A_2^1\left[T_{LE2}^1\right], \ldots, A_{r1}^1\left[T_{LEr1}^1\right]$	d_1
...	...	...
* $A^i\left[T_{LE}^i - cl\right]$	$A_1^i\left[T_{LE1}^i\right], A_2^i\left[T_{LE2}^i\right], \ldots, A_{r1}^i\left[T_{LEr1}^i\right]$	$d_i - cl$
...	...	...
* $A^m\left[T_{LE}^m - cl\right]$	$A_1^m\left[\overline{T}_{LE1}^m\right], A_2^m\left[\overline{T}_{LE2}^m\right], \ldots, A_{rm}^m\left[\overline{T}_{LErm}^m\right]$	d_m
...	...	...
$A^n\left[T_{LE}^n\right]$	$A_1^n\left[T_{LE1}^n\right], A_2^n\left[T_{LE2}^n\right], \ldots, A_{rn}^n\left[T_{LErn}^n\right]$	d_n

Table 4

where $\overline{T}_{LEj}^i$ is the new latest end time for the j^{th} predecessor of A^m. The project duration in terms of cl, (T_{cl}^P), becomes

$$T_{cl}^P = T_{LE}^m - cl \quad (5)$$

This duration is also equal to the project duration given by the second best critical path. The second best critical path is the critical path that is obtained after ignoring activity A^i. If the new project duration is given by T_{new}^P, then

$$T_{nor}^P - cl = T_{new}^P \quad (6)$$

i.e. $cl = T_{nor}^P - T_{new}^P$ (7)

When determining the new critical path, there is no need to start from the first node. Some of the activities are not affected by this change, and so they need not be used in the computations.

7. NUMERICAL ILLUSTRATION

The information given in Table 5 is used to answer the following questions.
By using activity latest end times, show that A^9 is critical, hence determine the project duration (T^P_{nor}) . Suppose activity A^9 is selected for crashing, compute the crash limit.

Activity	Predecessor	Duration (in days)
A^1	-	60
A^2	-	180
A^3	A^1	110
A^4	A^1	80
A^5	A^2	130
A^6	A^2	70
A^7	A^3	60
A^8	A^4, A^5	140
A^9	A^4, A^5	210
A^{10}	A^6	190
A^{11}	A^7, A^8	50
A^{12}	A^9, A^{10}	230

Table 5

7.1 Latest end times

The latest end times are generated as shown in Table 6. The critical activities are:

$$A^9 \leftarrow A^9 \leftarrow A^5 \leftarrow A^2 \quad (8)$$

Selection of the critical activities is done by starting from the bottom of Table 6.

The project duration is given by

$$T^P_{nor} = 750 \text{ days} \quad (9)$$

Activity	Predecessor	Duration (in days)
$A^{1}\begin{bmatrix}60\\ \end{bmatrix}$	-	60
* $A^{2}\begin{bmatrix}180\\ \end{bmatrix}$	-	180
$A^{3}\begin{bmatrix}170\\ \end{bmatrix}$	$A^{1}\begin{bmatrix}60\\ \end{bmatrix}$	110
$A^{4}\begin{bmatrix}140\\ \end{bmatrix}$	$A^{1}\begin{bmatrix}60\\ \end{bmatrix}$	80
* $A^{5}\begin{bmatrix}310\\ \end{bmatrix}$	$A^{2}\begin{bmatrix}180\\ \end{bmatrix}$	130
$A^{6}\begin{bmatrix}250\\ \end{bmatrix}$	$A^{2}\begin{bmatrix}180\\ \end{bmatrix}$	70
$A^{7}\begin{bmatrix}230\\ \end{bmatrix}$	$A^{3}\begin{bmatrix}170\\ \end{bmatrix}$	60
$A^{8}\begin{bmatrix}450\\ \end{bmatrix}$	$A^{4}\begin{bmatrix}140\\ \end{bmatrix}, A^{5}\begin{bmatrix}310\\ \end{bmatrix}$	140
* $A^{9}\begin{bmatrix}520\\ \end{bmatrix}$	$A^{4}\begin{bmatrix}140\\ \end{bmatrix}, A^{5}\begin{bmatrix}310\\ \end{bmatrix}$	210
$A^{10}\begin{bmatrix}450\\ \end{bmatrix}$	$A^{6}\begin{bmatrix}250\\ \end{bmatrix}$	190
$A^{11}\begin{bmatrix}500\\ \end{bmatrix}$	$A^{7}\begin{bmatrix}230\\ \end{bmatrix}, A^{8}\begin{bmatrix}450\\ \end{bmatrix}$	50
* $A^{12}\begin{bmatrix}750\\ \end{bmatrix}$	$A^{9}\begin{bmatrix}520\\ \end{bmatrix}, A^{10}\begin{bmatrix}450\\ \end{bmatrix}$	230

Table 6

7.2 Updating the latest end times

Table 7 is obtained by updating the latest end times.

Activity	Predecessor	Duration (in days)
$A^1\begin{bmatrix}60\\ \end{bmatrix}$	-	60
* $A^2\begin{bmatrix}180\\ \end{bmatrix}$	-	180
$A^3\begin{bmatrix}170\\ \end{bmatrix}$	$A^1\begin{bmatrix}60\\ \end{bmatrix}$	110
$A^4\begin{bmatrix}310\\ \end{bmatrix}$	$A^1\begin{bmatrix}60\\ \end{bmatrix}$	80
* $A^5\begin{bmatrix}310\\ \end{bmatrix}$	$A^2\begin{bmatrix}180\\ \end{bmatrix}$	130
$A^6\begin{bmatrix}250\\ \end{bmatrix}$	$A^2\begin{bmatrix}180\\ \end{bmatrix}$	70
$A^7\begin{bmatrix}450\\ \end{bmatrix}$	$A^3\begin{bmatrix}170\\ \end{bmatrix}$	60
$A^8\begin{bmatrix}450\\ \end{bmatrix}$	$A^4\begin{bmatrix}310\\ \end{bmatrix}, A^5\begin{bmatrix}310\\ \end{bmatrix}$	140
* $A^9\begin{bmatrix}520\\ \end{bmatrix}$	$A^4\begin{bmatrix}310\\ \end{bmatrix}, A^5\begin{bmatrix}310\\ \end{bmatrix}$	210
$A^{10}\begin{bmatrix}450\\ \end{bmatrix}$	$A^6\begin{bmatrix}250\\ \end{bmatrix}$	190
$A^{11}\begin{bmatrix}500\\ \end{bmatrix}$	$A^7\begin{bmatrix}450\\ \end{bmatrix}, A^8\begin{bmatrix}450\\ \end{bmatrix}$	50
* $A^{12}\begin{bmatrix}750\\ \end{bmatrix}$	$A^9\begin{bmatrix}520\\ \end{bmatrix}, A^{10}\begin{bmatrix}520\\ \end{bmatrix}$	230

Table 7

7.3 Earliest end times

The earliest end times are generated as shown in Table 8.

Activity	Predecessor	Duration (in days)
$A^1\begin{bmatrix}60\\230\end{bmatrix}$	-	60
* $A^2\begin{bmatrix}180\\180\end{bmatrix}$	-	180
$A^3\begin{bmatrix}170\\640\end{bmatrix}$	$A^1\begin{bmatrix}60\\230\end{bmatrix}$	110
$A^4\begin{bmatrix}310\\310\end{bmatrix}$	$A^1\begin{bmatrix}60\\230\end{bmatrix}$	80
* $A^5\begin{bmatrix}310\\310\end{bmatrix}$	$A^2\begin{bmatrix}180\\180\end{bmatrix}$	130
$A^6\begin{bmatrix}250\\330\end{bmatrix}$	$A^2\begin{bmatrix}180\\180\end{bmatrix}$	70
$A^7\begin{bmatrix}450\\700\end{bmatrix}$	$A^3\begin{bmatrix}170\\640\end{bmatrix}$	60
$A^8\begin{bmatrix}450\\700\end{bmatrix}$	$A^4\begin{bmatrix}310\\310\end{bmatrix}, A^5\begin{bmatrix}310\\310\end{bmatrix}$	140
* $A^9\begin{bmatrix}520\\520\end{bmatrix}$	$A^4\begin{bmatrix}310\\310\end{bmatrix}, A^5\begin{bmatrix}310\\310\end{bmatrix}$	210
$A^{10}\begin{bmatrix}520\\520\end{bmatrix}$	$A^6\begin{bmatrix}250\\330\end{bmatrix}$	190
$A^{11}\begin{bmatrix}500\\750\end{bmatrix}$	$A^7\begin{bmatrix}450\\700\end{bmatrix}, A^8\begin{bmatrix}450\\700\end{bmatrix}$	50
* $A^{12}\begin{bmatrix}750\\750\end{bmatrix}$	$A^9\begin{bmatrix}520\\520\end{bmatrix}, A^{10}\begin{bmatrix}520\\520\end{bmatrix}$	230

Table 8

From Table 6 the critical activity A^9, selected for crashing, is reduced by cl, and recalculations are done in terms of cl as presented in Table 9.

Activity	Predecessor	Duration (in days)
$A^1\begin{bmatrix}60\\ \end{bmatrix}$	-	60
* $A^2\begin{bmatrix}180\\ \end{bmatrix}$	-	180
$A^3\begin{bmatrix}170\\ \end{bmatrix}$	$A^1\begin{bmatrix}60\\ \end{bmatrix}$	110
$A^4\begin{bmatrix}140\\ \end{bmatrix}$	$A^1\begin{bmatrix}60\\ \end{bmatrix}$	80
* $A^5\begin{bmatrix}310\\ \end{bmatrix}$	$A^2\begin{bmatrix}180\\ \end{bmatrix}$	130
$A^6\begin{bmatrix}250\\ \end{bmatrix}$	$A^2\begin{bmatrix}180\\ \end{bmatrix}$	70
$A^7\begin{bmatrix}230\\ \end{bmatrix}$	$A^3\begin{bmatrix}170\\ \end{bmatrix}$	60
$A^8\begin{bmatrix}450\\ \end{bmatrix}$	$A^4\begin{bmatrix}140\\ \end{bmatrix}, A^5\begin{bmatrix}310\\ \end{bmatrix}$	140
* $A^9\begin{bmatrix}520-cl\\ \end{bmatrix}$	$A^4\begin{bmatrix}140\\ \end{bmatrix}, A^5\begin{bmatrix}310\\ \end{bmatrix}$	210-*cl*
$A^{10}\begin{bmatrix}450\\ \end{bmatrix}$	$A^6\begin{bmatrix}250\\ \end{bmatrix}$	190
$A^{11}\begin{bmatrix}500\\ \end{bmatrix}$	$A^7\begin{bmatrix}230\\ \end{bmatrix}, A^8\begin{bmatrix}450\\ \end{bmatrix}$	50
* $A^{12}\begin{bmatrix}750-cl\\ \end{bmatrix}$	$A^9\begin{bmatrix}520-cl\\ \end{bmatrix}, A^{10}\begin{bmatrix}450\\ \end{bmatrix}$	230

Table 9

The project duration (T_{cl}^{P}), which is in terms of cl, is given by

$$T_{nor}^{P} = (750 - cl) \text{ days} \tag{10}$$

The second-best critical activities and new project duration are determined by ignoring activity A^9, as presented in Table 10.

Activity	**Predecessor**	**Duration (in days)**
$A^{1}\begin{bmatrix}60\\ \ \end{bmatrix}$	-	60
* $A^{2}\begin{bmatrix}180\\ \ \end{bmatrix}$	-	180
$A^{3}\begin{bmatrix}170\\ \ \end{bmatrix}$	$A^{1}\begin{bmatrix}60\\ \ \end{bmatrix}$	110
$A^{4}\begin{bmatrix}140\\ \ \end{bmatrix}$	$A^{1}\begin{bmatrix}60\\ \ \end{bmatrix}$	80
$A^{5}\begin{bmatrix}310\\ \ \end{bmatrix}$	$A^{2}\begin{bmatrix}180\\ \ \end{bmatrix}$	130
* $A^{6}\begin{bmatrix}250\\ \ \end{bmatrix}$	$A^{2}\begin{bmatrix}180\\ \ \end{bmatrix}$	70
$A^{7}\begin{bmatrix}230\\ \ \end{bmatrix}$	$A^{3}\begin{bmatrix}170\\ \ \end{bmatrix}$	60
$A^{8}\begin{bmatrix}450\\ \ \end{bmatrix}$	$A^{4}\begin{bmatrix}140\\ \ \end{bmatrix}, A^{5}\begin{bmatrix}310\\ \ \end{bmatrix}$	140
~~$A^{9}\begin{bmatrix}520\\ \ \end{bmatrix}$~~	$A^{4}\begin{bmatrix}140\\ \ \end{bmatrix}, A^{5}\begin{bmatrix}310\\ \ \end{bmatrix}$	~~210~~
* $A^{10}\begin{bmatrix}450\\ \ \end{bmatrix}$	$A^{6}\begin{bmatrix}250\\ \ \end{bmatrix}$	190
$A^{11}\begin{bmatrix}500\\ \ \end{bmatrix}$	$A^{7}\begin{bmatrix}230\\ \ \end{bmatrix}, A^{8}\begin{bmatrix}450\\ \ \end{bmatrix}$	50
* $A^{12}\begin{bmatrix}670\\ \ \end{bmatrix}$	~~$A^{9}\begin{bmatrix}520\\ \ \end{bmatrix}$~~, $A^{10}\begin{bmatrix}450\\ \ \end{bmatrix}$	230

Table 10

The new project duration (T^{P}_{new}), which is also the duration for the second-best critical path, is given by

$$T^{P}_{new} = 670 \text{ days} \tag{11}$$

$$cl = T^{P}_{nor} - T^{P}_{new} = 750 - 670 = 80 \text{ days} \tag{12}$$

Note: The only activities that are used in crashing activity A^9, out of the 12 given activities, are A^{10} and A^{12}. The rest of the activities are not affected.

8. CONCLUSIONS

The procedure presented in this paper determines critical activities, critical path, start and end times, free floats, crash limits, and all other useful information without the use of a network model. The project network diagram model is not easy to draw, and in any case is not necessary. All the information that is required for decision-making in project management may be efficiently extracted from the available data without the use of a network model. When computing the crash limit, only a fraction of the activities is required. Calculations do not necessarily have to start from the first activities, and the new start and end times are always readily available.

9. REFERENCES

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