S A Journal of Industrial Engineering, Vol. 1, No. 1, June 1987, pp. 12-22

RELIABILITY ANALYSIS USING SIMULATION MODELLING

S J CLAASEN PrIng
and
P S KRUGER

Department of Industrial and Systems Engineering
University of Pretoria, Pretoria, South Africa

ABSTRACT

The reliability analysis of complex systems may often become unmanageable, especially when state or time dependent failure rates, repair facilities and standby operations are present in a system. This paper describes the possible use of a simulation approach and the development of a reliability, availability and maintainability simulator which may be used to alleviate some of the disadvantages inherent in the traditional analytical approach.

OPSOMMING

Die analise van die betroubaarheid van 'n komplekse stelsel mag soms onhanteerbaar raak veral indien die stelsel toestand- of tydafhanklike falingstempo's, herstelfasiliteite en bystandtoerusting insluit. Hierdie artikel beskryf die moontlike toepassing van 'n simulasiebenadering asook die ontwikkeling van 'n betroubaarheid-, beskikbaarheid- en instandhoubaarheid-simulator wat gebruik mag word om sommige van die nadele, inherent aan die tradisionele analitiese benadering, te oorkom.

INTRODUCTION

The reliability analysis of complex systems may often become unmanageable [1]. This is especially true when state and time dependent failure rates, repair facilities and standby operations are present in a system. Shooman [4] describes three different approaches to reliability computations for systems involving such cases :

* An approach based on the use of Markov models,
* an approach based on the use of joint density functions, and
* an approach based on the use of convolution like integrations.

The Markov model approach works well and has much appeal as long as the failure and repair rates are constant, which severely limits the possible applications. If this is not the case this method is no longer applicable except in a few special instances.

The joint density function and the convolution like integration approaches are still valid when the failure or repair rates are time-dependent. However, the implied mathematical sophistication of the user and the degree of difficulty involved makes it highly unlikely that these approaches will be applied on a regular basis in the average industrial environment.

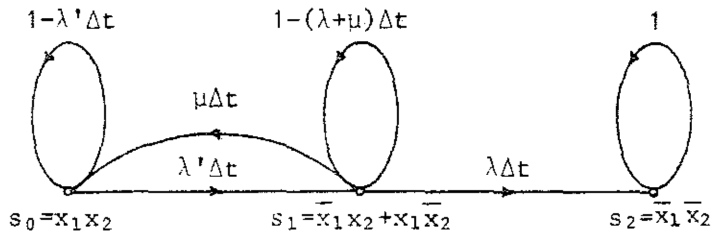
Reliability analysis of complex systems using a simulation modelling approach may eliminate most of the abovementioned disadvantages while at the same time it may provide additional information usually not available when employing the analytical approach. Furthermore, a general purpose and widely applicable reliability simulator may be designed in the form of a software package which may be used effectively by a user who need not necessarily be a reliability expert.

AN ILLUSTRATIVE EXAMPLE

For the purpose of demonstrating the difficulties involved in the analytical approach and the possible advantages of the simulation approach, consider a relatively simple example consisting of a two component parallel subsystem with repair.

THE MARKOV MODEL APPROACH

Shooman [4] solves this problem analytically by means of the Markov model approach. The Markov reliability model is shown in Figure 1.



$\lambda' = 2\lambda$

λ = constant failure rate for both components

μ = constant repair rate for both components

Figure 1. Markov reliability model for two identical parallel elements and one repair facility.

The differential equations associated with figure 1 are :

$$\dot{P}_{s_0}(t) + \lambda P_{s_0}(t) = \mu P_{s_1}(t)$$
$$\dot{P}_{s_1}(t) + (\mu + \lambda)P_{s_1}(t) = \lambda' P_{s_0}(t)$$
$$\dot{P}_{s_2}(t) = \lambda P_{s_1}(t)$$
$$P_{s_0}(0) = 1 \qquad P_{s_1}(0) = P_{s_2}(0) = 0$$

Taking the Laplace transform of this set of equations yields :

$$(s + \lambda')P_{s_0}(s) - \mu P_{s_1}(s) + 0P_{s_2}(s) = 1$$
$$-\lambda' P_{s_0}(s) + (s + \mu + \lambda)P_{s_1}(s) + 0P_{s_2}(s) = 0$$
$$0P_{s_0}(s) - \lambda P_{s_1}(s) + (s)P_{s_2}(s) = 0$$

Solution via Cramer's rule yields :

$$P_{s_0}(s) = \frac{(s + \lambda + \mu)}{s^2 + (\lambda + \lambda' + \mu)s + \lambda\lambda'} \qquad \text{..........(1)}$$

$$P_{s_1}(s) = \frac{\lambda'}{s^2 + (\lambda + \lambda' + \mu)s + \lambda\lambda'} \qquad \text{..........(2)}$$

$$P_{s_2}(s) = \frac{\lambda\lambda'}{s\{s^2 + (\lambda + \lambda' + \mu)s + \lambda\lambda'\}} \qquad \text{..........(3)}$$

The solution for the roots of the denominator quadratic (the system poles) yields :

$$r_1, r_2 = \frac{-(\lambda + \lambda' + \mu) \pm \sqrt{(\lambda + \lambda' + \mu)^2 - 4\lambda\lambda'}}{2}$$

Expanding equations (1), (2) and (3) into partial fractions yields :

$$P_{s_0}(s) = \frac{(s + \lambda + \mu)}{(s - r_1)(s - r_2)} = \frac{(\lambda + \mu + r_1)/(r_1 - r_2)}{s - r_1} + \frac{(\lambda + \mu + r_2)/(r_2 - r_1)}{s - r_2}$$

$$P_{s_1}(s) = \frac{\lambda'}{(s - r_1)(s - r_2)} = \frac{\lambda'/(r_1 - r_2)}{s - r_1} + \frac{\lambda'/(r_2 - r_1)}{s - r_2}$$

$$P_{s_2}(s) = \frac{\lambda\lambda'}{s(s - r_1)(s - r_2)} = \frac{\lambda\lambda'/r_1 r_2}{s} + \frac{\lambda\lambda'/r_1(r_1 - r_2)}{s - r_1} + \frac{\lambda\lambda'/r_2(r_2 - r_1)}{s - r_2}$$

The inverse transform yields :

$$P_{s_0}(t) = \frac{\lambda + \mu + r_1}{r_1 - r_2} e^{r_1 t} - \frac{\lambda + \mu + r_2}{r_1 - r_2} e^{r_2 t}$$

$$P_{s_1}(t) = \frac{\lambda'}{r_1 - r_2} e^{r_1 t} - \frac{\lambda'}{r_1 - r_2} e^{r_2 t}$$

$$P_{s_2}(t) = 1 + \frac{r_2}{r_1 - r_2} e^{r_1 t} - \frac{r_1}{r_1 - r_2} e^{r_2 t}$$

The system reliability is given by $Ps_0(t) + Ps_1(t)$. Solving this expression for $\lambda = 0{,}02$ and $\mu = 0{,}1$ (single repair) yields :

$$R(t) = 1{,}0345\ e^{r_1 t} - 0{,}0345\ e^{r_2 t}$$

Integrating $R(t)$ between zero and infinity yields the mean time-between-failures which is 200 time units.

THE SIMULATION APPROACH

The same r-out-of-n subsystem with repair may be analyzed by using a simulation approach. Although such a simulation model may be designed using a general purpose computer language such as FORTRAN or PASCAL a variety of powerful simulation languages exist which may make the modelling task significantly easier [2]. The SLAM simulation language [3] will be used for demonstration purposes and the SLAM network used to model the r-out-of-n subsystem with repair is shown in figure 2. The model shown in figure 2 forms part of an extended SLAM model (RAMSIM) with the capability of modelling a variety of different subsystems.

Description of the SLAM network model

A single entity is created at the CREATE node C1. If the user indicates the presence of an r-out-of-n subsystem with repair, XX(8) takes on the value of 1. At node G40 the value of n (represented by XX(9)) determines the number of component lifetimes generated. On the failure of a component the number of surviving components is calculated at the ASSIGN node A44. If the number of surviving components is less than r the system fails. Otherwise the failed component goes to the AWAIT node AW40 where, depending on the availability of a repair facility, it is repaired. Upon repair the system is again checked for failure. If the system is not in the failed state the repaired component is placed in position again and continues operation. When the system fails the time to failure is collected at the COLLECT node COL.

Simulation model results

Running the system to accumulate 200 system failures, resulted in a mean time-between-failures of 206 time units which compares favourably with the true mean time-between-failures of 200 time units. The simulated and true reliability for the subsystem is compared in figure 3.

Other output obtainable from the simulation run includes :

* A histogram of time-between-failures,

* the average utilization of the repair facilities, and

* the average number of repairs per component before system failure.

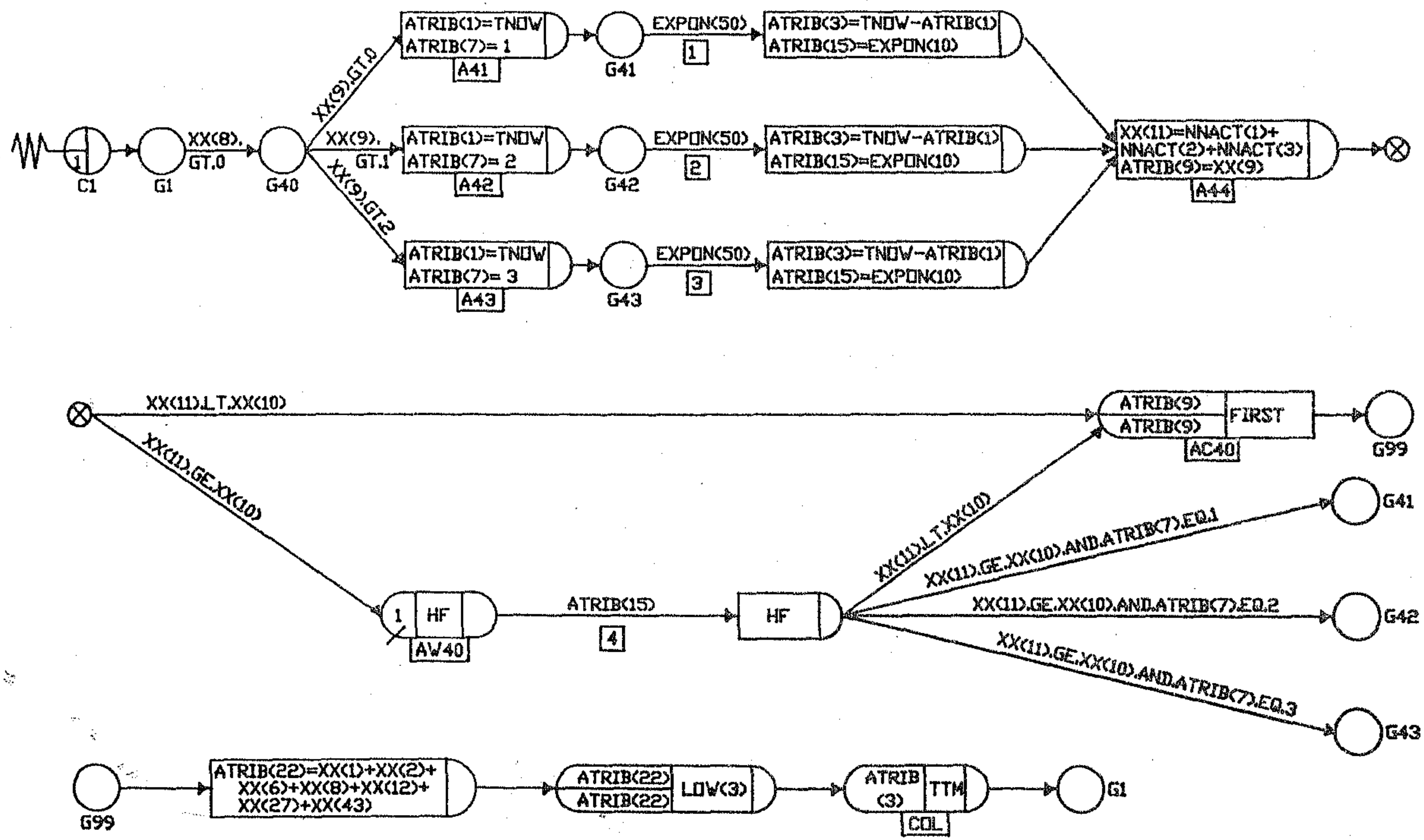


Figure 2. SLAM Network for an r-out-of-3 subsystem with repair.

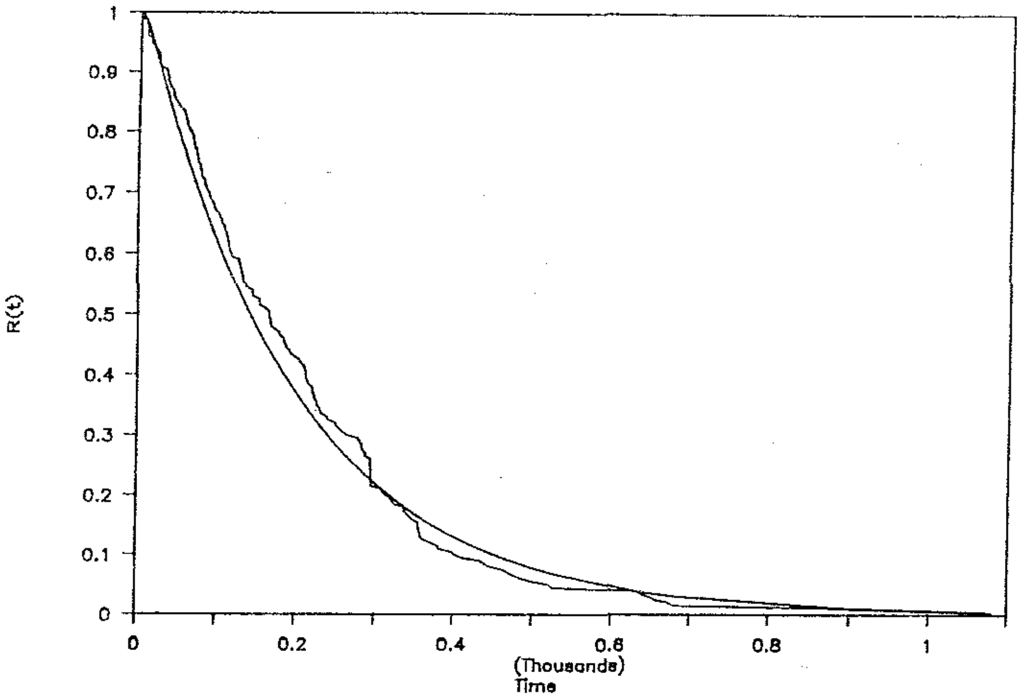


Figure 3. Comparison of simulated and true reliability for the two component parallel subsystem with repair.

Extending the model

Although the model in figure 2 can only handle one r-out-of-n subsystem with user specified values for n and r (n less than or equal to three in this model) and with one repair facility, extending the model is a relatively simple task from a conceptual point of view. This is illustrated in figure 4 which depicts the SLAM model of figure 2, extended to accommodate n components and also showing the interfaces with the rest of an extended model.

The model may be further enhanced to include :

* Any user specified distribution for the time between failures of components,

* any user specified distribution for the repair time of components,

* any user specified values of n and r, and

* more than one repair facility.

Furthermore, a variety of other types of reliability subsystems may be included in the model thus providing a general purpose reliability simulator.

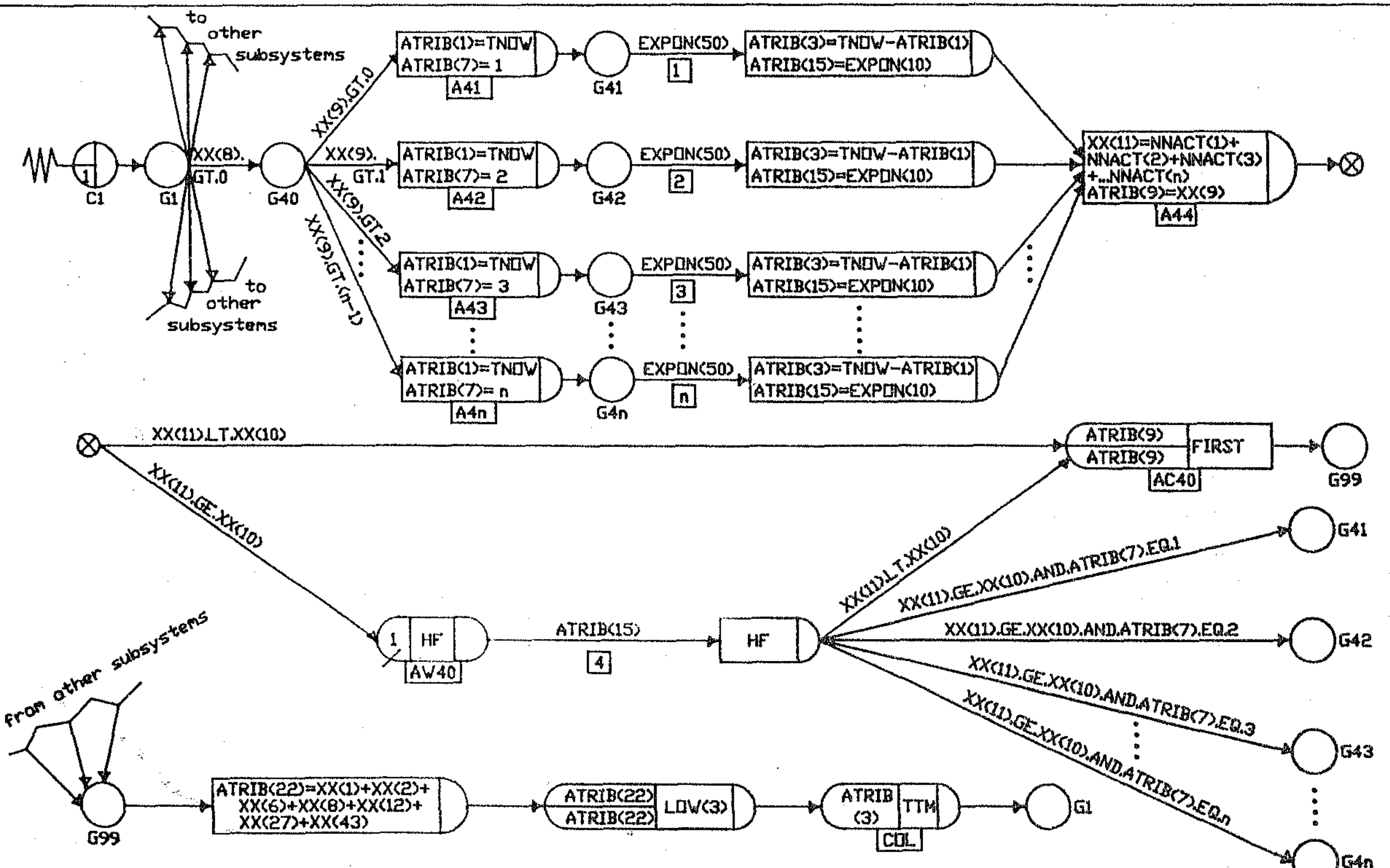


Figure 4. SLAM Network for an r-out-of-n subsystem with repair.

RAMSIM SIMULATOR

As indicated in the previous paragraph it should be possible to develop a general purpose Reliability, Availability and Maintainability Simulator (RAMSIM) and such a simulator is presently under development. RAMSIM is a menu-driven simulator, developed for use on a microcomputer and is based on the SLAM simulation language [3].

RAMSIM requires the user to define the system (of which the reliability or availability is to be determined) in terms of the following subsystems :

* Series subsystem,
* r-out-of-n subsystem,
* standby subsystem,
* r-out-of-n subsystem with repair,
* standby subsystem with repair,
* r-out-of-n subsystem with dependence, and
* r-out-of-n subsystem with dependence and repair.

RAMSIM can evaluate any of these seven types of subsystems individually or a system consisting of any combination of these subsystems combined in a series configuration.

RAMSIM requires the following information for system definition :

* The types of subsystems in the system,
* the number of components in each subsystem,
* the number of components needed for subsystem success (r-out-of-n subsystems only),
* the time to failure distribution and distribution parameters for each component,
* the time to repair distribution and distribution parameters for each component (repairable subsystems only),

* the number of repair facilities available, and

* the time to failure distribution and distribution parameters for each component for each system state (dependent subsystems only).

Certain limitations have been placed on the maximum values of the system parameters and therefore on the maximum system complexity that can be handled. The extent of these limitations will be determined by the capacity of the computer equipment being used and to a lesser degree by the constraints inherent in the particular version of the SLAM software. At the present state of development of RAMSIM it seems reasonable to assume that the use of a typical microcomputer and the standard microcomputer version of SLAM will be adequate to handle reliability systems with a significant degree of complexity.

CONCLUSIONS

The possible advantages of the simulation approach may be summarized as follows :

* The user is not involved in cumbersome mathematical model formulation and manipulation but only needs to define the system configuration as detailed above,

* the analysis is not limited to constant failure or repair rates,

* the ease of use of the simulator lends itself to answering "what if" type questions. This should make the simulator a valuable tool for reliability and availability calculations of various system configurations and repair scenarios (including different degrees of maintainability), during system design and logistics planning, and

* the simulation modelling approach may provide additional information usually not available when employing the analytical approach such as :

  - The average utilization of repair facilities,

  - the average number of repairs per component before system failure,

  - the mean time to repair for the system,

- the maximum corrective maintenance time, and
- the mean down time.

The possible disadvantages of the simulation approach may be summarized as follows :

* Since the simulation approach is a sampling procedure all the results obtained are only estimates of the true values and should be interpreted as such. It is therefore necessary to pay particular attention to the problems of stochastic stability in the design of such a simulator.

* The use of the simulator is restricted to the subsystem and system configurations described above. The possibility of providing the user with the capability to combine the defined subsystems in any system configuration is under investigation.

At present RAMSIM uses the standard SLAM summary report output format but a specialized menu-driven reliability and availability report generator will be developed.

==================

REFERENCES

[1] CLAYTON E R and COOLEY J W, "Use of Q-GERT Network Simulation in Reliability Analysis," IEEE Transactions on Reliability, vol R-30, no 4, Oct 1981, pp 321-324.

[2] KRUGER P S, "Micro-Computer Simulation Software : A Review", ORiON, Vol. 2, No. 1, pp 1-14, 1986.

[3] PRITSKER A A B, "Introduction to Simulation and SLAM II", John Wiley and Sons, 1984.

[4] SHOOMAN M L, "Probabilistic Reliability: An Engineering Approach", McGraw-Hill, 1968.