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THE KNAPSACK PROBLEM REVISISTED - A SIMPLE AND EFFECTIVE TOOL FOR INDUSTRIAL DECISION-MAKING AT ALL LEVELS

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The knapsack problem is a classical optimization problem in which an optimum set of items is chosen according to some or other attribute, and subject to a limiting constraint(bottleneck).
The problem is often solved using integer linear programming software. However if the number of items is not large, it can be solved very simply, and very efficiently using graphical methods.
The problem is a common one in industry, occuring at all levels in the organization. The graphical technique for the knapsack problem is excellent for examining these problems and is a tool which could be used more often with good results.

OPSOMMING

Die bekende „Knapsack Problem" is 'n oorspronklike lineêre programmerings model vir die keuse van 'n optimaal versameling items uit 'n groter bevolking, op grond van een of ander eienskap, en onderhewig aan sekere beperkings.
Die' probleem word gewoonlik deur gebruik van rekenaar programmatuur opgelos, maar indien die aantal items klein genoeg is om hanteerbaar te wees, is dit makliker om grafiese metodes te gebruik.
Vir die oplossing van probleme van „Knapsack" aard, wat dikwels in die bedryfsomgewing voorkom, is grafiese metodes van uiters belang, en behoort meer aandag te geniet.

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INTRODUCTION

The knapsack problem was originally introduced by George Dantzig [1] in 1963.
The problem concerns a hiker who has a number of items of camping equipment he wishes to take with him on a trip.
Each item has a certain value to the hiker, and each has a certain weight. The hiker cannot take them all as this will exceed the weight he wishes to restrict his backpack to.
What he must do is to find the combination of items which will meet his weight restriction, and provide the maximum value to him.

The statement of the problem in linear programming terms is:

MAXIMIZE $V = \Sigma v_j x_j$

SUBJECT TO:

$\Sigma w_j x_j \leqslant W$(the maximum load)

$x_j = 1$ or 0

WHERE:

V is the total value of the selected items
v_j is the value assigned to the jth item
w_j is the weight of the jth item
x_j is a variable which takes on the value zero or one.

The graphical solution to the problem is very simple - yet effective.

The method is to construct a graph as shown in Figure 1 with value represented on the vertical-axis and weight on the horizontal-axis.
The ray 'r' is swept out in a clockwise direction and this automatically selects the 'best' items to take, one by one, until the situation is reached where:

$$w_1x_1 + w_2x_2 + \ldots \; w_kx_k \geqslant W$$

AND

$$w_1x_1 + w_2x_2 + \ldots \; w_{k-1}x_{k-1} < W$$

If the equality holds then the optimum solution has been obtained.

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Otherwise, the ray is kept stationary and items in the immediate vicinity of the ray are examined for inclusion, working from the extremity of the ray, to the origin.

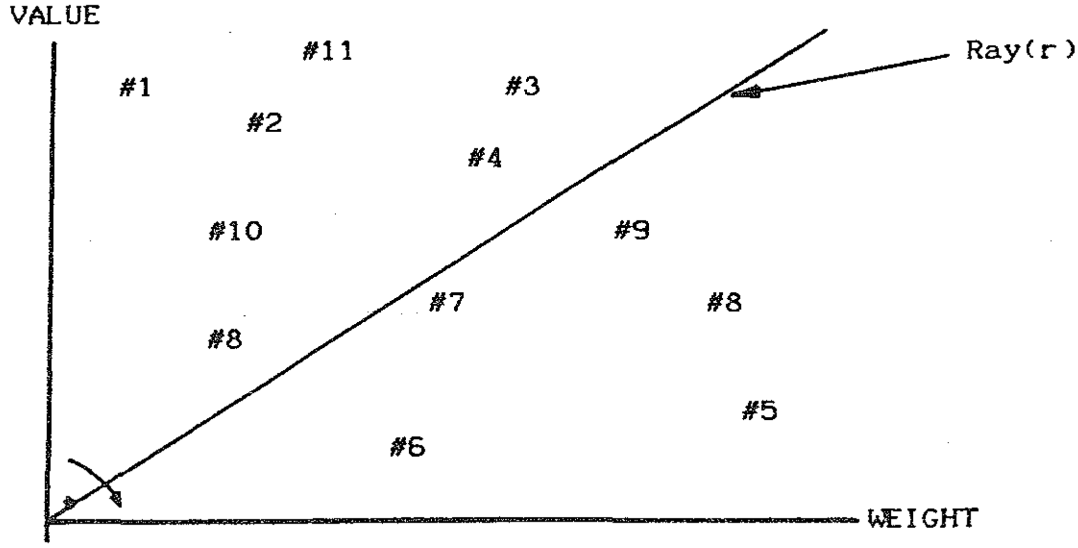

FIGURE 1: Graphical Optimization of Total Value (V)
$w_1 + w_2 + w_3 + w_4 + w_8 + w_{10} + w_{11} = W$

INDUSTRIAL APPLICATIONS

- The first application which comes to mind occurs at a high level in the organizational hierarchy. This involves the allocation of a limited budget to a number of competing options. The problem is that the budget cannot enable all the options to be realised.
  In this case, the value of each option must be decided in financial or in subjective terms, or a combination of both.

  Their values will in effect be a measure of their relative importance to the company.
  This value is plotted on the vertical-axis and the actual cost of the option is plotted on the horizontal-axis.
  The optimum selection of options can then be obtained exactly as described above for the knapsack problem.
  The ray is swept out in a clockwise direction and the costs accumulated until the budget figure is reached.
  The selection of options thus obtained is the optimal set.

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- Lower down in the decision-making hierarchy, there is always the problem of classifying inventory, or components, or customer orders, etc. into categories which reflect their criticality or importance.
  For example, in procurement situations the familiar A, B, C classification is used.
  If the cost of the item is plotted on the vertical-axis and the usage is plotted on the horizontal-axis with the highest usage at the origin, and low usage further out, then the ray will select items according to their importance as shown in Figure 2.

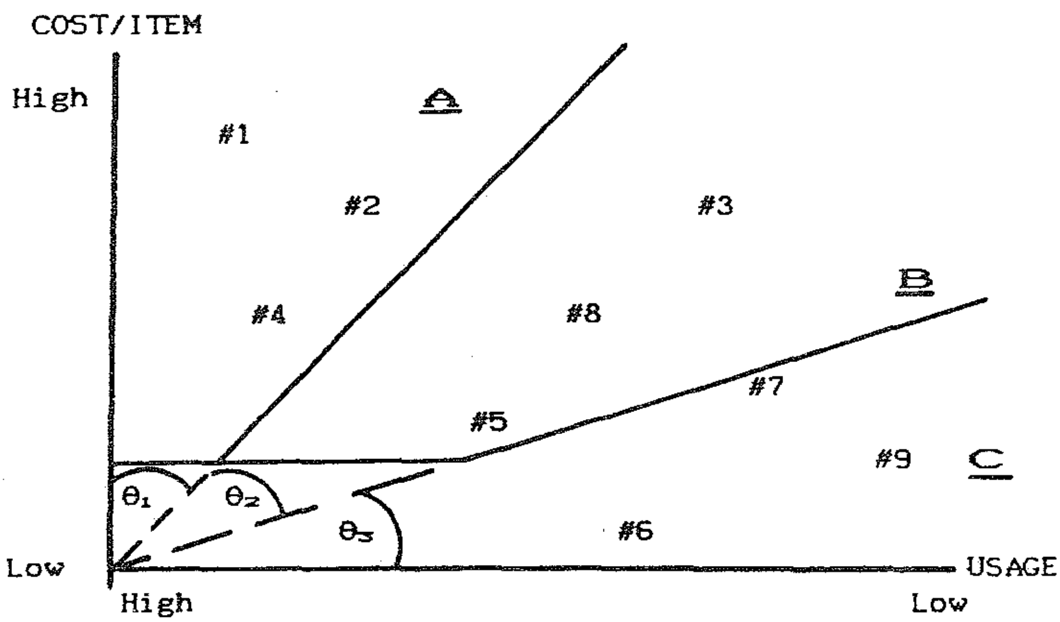

Figure 2: A B C Classifiacation of Inventory Items

The values of the angles θ_1, θ_2, and θ_3 must be chosen accordingly, - they need not all be 30degrees.

- When planning for the acquisition of components(at MRP level), the variables of importance may the LEAD TIME, and the AVAILABILITY of the component.
  For this situation, the lead time is plotted on the vertical-axis and the number of suppliers(from 1 at the origin, increasing further out) is plotted on the horizontal-axis.
  The ray then classifies the components as it sweeps.

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- Scheduling is another area of application. In this case the variables may be TIME REMAINING TO COMPLETION OF ORDER OR JOB, and CRITICAL RATIO[defined as (time to due date)/(time to completion)].
  The first of these is plotted on the vertical-axis, and the second is plotted on the horizontal-axis with low CR's near the origin, large CR's further out.

  The ray will highlight the most critical jobs first, those with low CR's and long times to completion. It will also automatically sort the high CR jobs by time to completion, thus providing the scheduler with a priority listing.
  This situation is depicted in Figure 3.

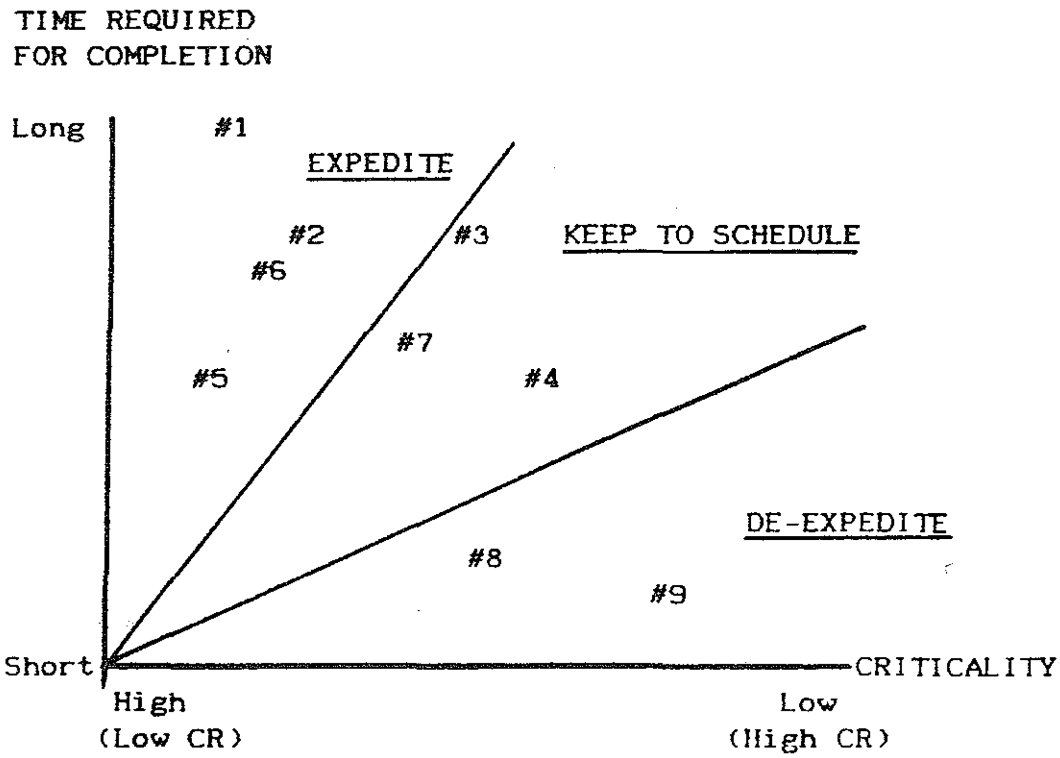

Figure 3: Establishing Job Priorities

- Finally, quality is yet another area of interest for the application of this technique. The vertical-axis could describe customer complaints relating to particular aspects of quality, and the horizontal-axis the cost of rectifying the quality of these aspects .
  The ray will provide an A, B, C classification with

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class A items being the ones which cause the most complaints, but are the easiest to fix.

A common present-day application is to classify products according to their average yield(in terms of some parameter), and the signal-to-noise ratio(S/N ratio) determined by varying environmental factors. This ratio measures the robustness of the product design. Usually, the idea is to obtain a high S/N ratio, and a high yield simultaneously in a product.
This situation is depicted in Figure 4.

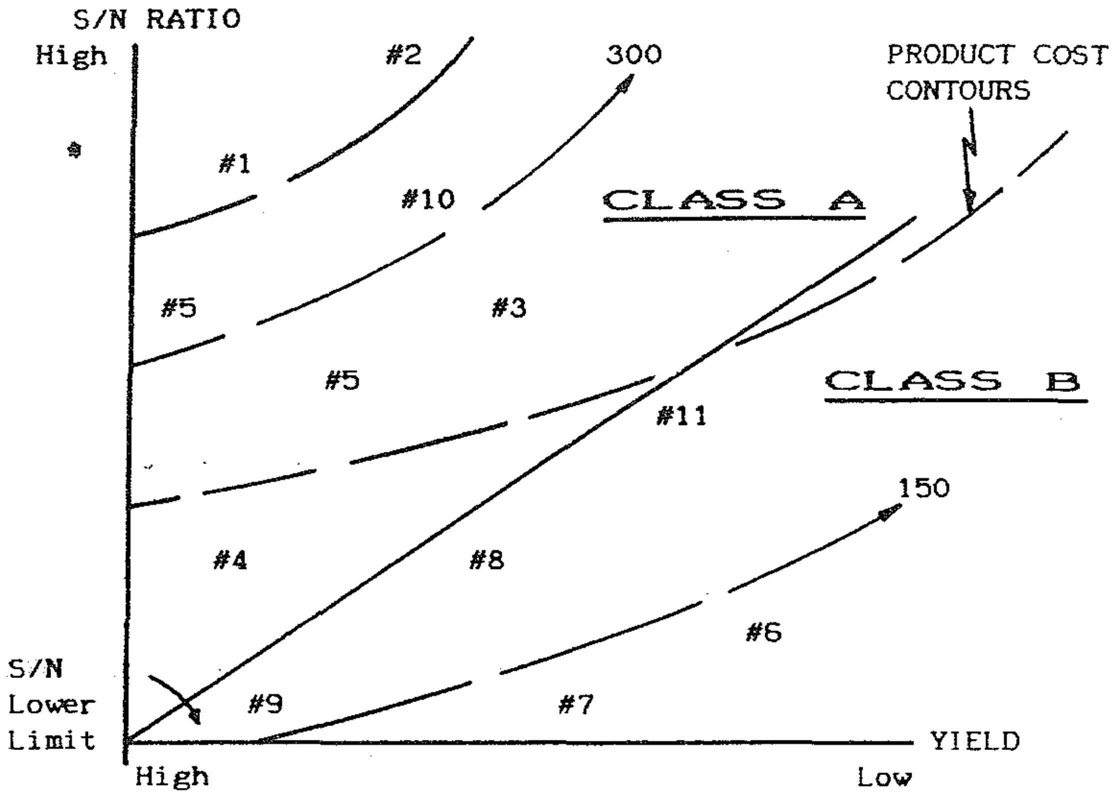

Figure 4: Product Quality

The application of the knapsack problem is almost limitless. One other area, not mentioned above is that of isolating bottlenecks. With a suitable choice of variables we can classify the bottlenecks in the organization into A, B, or C, and work to improve them in this order.

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CONCLUSION

The knapsack problem can be formulated and solved as an integer linear programming problem.
However for small problems, and for managers without a mathematical/statistical background, the simple graphical approach is very effective, and provides the optimum solution in terms of the variables selected for the axes.
Its use is virtually unlimited.
As an extension, it is possible to program the method as a spreadsheet application using LOTUS 1-2-3 or a similar package.

The main point is that it forces managers to be clear about the variables they are using, how these variables relate to one another, and what the job(etc.) priorities are, in terms of these variables.

REFERENCES

1. DANTZIG G.B. Linear Programming and Extensions, Princeton University Press, Princeton, New Jersey, (1963).