A Comparative Study of Deterministic and Stochastic Programming Approaches to Optimise Marketing Campaigns

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ABSTRACT

Many factors continually influence customer behavioural patterns, leading to complex and uncertain decision-making. Traditional deterministic approaches, while useful, do not account for uncertainty, potentially causing suboptimal decisions and reduced profitability. Retailers should adopt models that incorporate uncertainty while maximising profitability to address this. This paper proposes a two-stage stochastic programming model, referred to as a recourse model, to optimise a marketing campaign's profitability while providing solutions that hedge against uncertainty. A deterministic counterpart is also proposed, based on two existing deterministic models from the literature. The main contribution of this study involves the formulation of the recourse model and the added value of a stochastic approach in handling uncertainty more effectively. The proposed recourse model opens up new frontiers for marketing campaign optimisation by incorporating demand uncertainty and providing decisions that hedge against uncertainty.

OPSOMMING

Baie faktore beïnvloed voortdurend kliënte se gedragspatrone, wat lei tot komplekse en onsekere besluitneming. Tradisionele deterministiese benaderings, hoewel nuttig, hou nie rekening met onsekerheid nie, wat moontlik suboptimale besluite en verminderde winsgewendheid kan veroorsaak. Kleinhandelaars moet modelle aanneem wat onsekerheid insluit terwyl hulle winsgewendheid maksimeer om dit aan te spreek. Hierdie artikel stel 'n twee-stadium stogastiese programmeringsmodel voor, waarna verwys word as 'n verhaalmodel, om die winsgewendheid van 'n bemarkingsveldtog te optimaliseer terwyl oplossings gebied word wat onsekerheid beskerm. 'n Deterministiese eweknie word ook voorgestel, gebaseer op twee bestaande deterministiese modelle uit die literatuur. Die hoofbydrae van hierdie studie behels die formulering van die verhaalmodel en die toegevoegde waarde van 'n stogastiese benadering om onsekerheid meer effektief te hanteer. Die voorgestelde verhaalmodel open nuwe grense vir bemarkingsveldtogoptimalisering deur vraagonsekerheid in te sluit en besluite te verskaf wat onsekerheid beskerm.

1. INTRODUCTION

A marketing campaign involves directed efforts to promote a product or service to a specific target audience, typically within a designated timeframe [1]. Successful campaigns rely on targeting the right audience with the right product at the right time [2].

Pepsi's 2014 summer digital campaign in Turkey strengthened its market share by 5% through personalised offers and mobile engagement [3]. Levi Strauss & Co.'s 2009 iPhone-targeted ad campaign illustrated the value of understanding the characteristics of the targeted audience and providing interactive content. These examples, presented by Ryan [3], show the importance of customer-centric marketing and effective allocation strategies. Retailers often use such campaigns to increase revenue by encouraging repeat customer purchases.

However, optimising a marketing campaign is a complex problem, particularly under demand uncertainty, which involves the unpredictability of customer behaviour and preferences. Dyer *et al.* [4] stated that consumer behaviour is influenced by numerous external factors, making it difficult for retailers to predict purchasing patterns. Uncertainty, described as a lack of knowledge about future outcomes [5], is a key problem when aligning marketing actions with customer responses.

In operations research, optimisation approaches are broadly categorised as deterministic or stochastic. Deterministic optimisation assumes that all variables are known or estimated from historical data, leading to decisions based on a single expected scenario [6]. In contrast, optimisation under uncertainty, such as stochastic programming (SP), considers randomness and hedges decisions against uncertainty [7]. SP problems, particularly two-stage stochastic programming models (recourse models), allow decision-makers to make initial "here-and-now" decisions that remain adaptable once uncertainty has been revealed. While more computationally intensive, these models offer enhanced robustness and realistic planning [8].

Although numerous deterministic models for campaign optimisation exist, such as the integer linear programming (ILP) models of Lin [5] and Hellemo *et al.* [9], these rely solely on historical data and fail to hedge against demand uncertainty. As a result, the solutions may become infeasible or suboptimal if actual customer behaviour diverges from assumptions [10]. Thus, this study proposes a two-stage stochastic programming model that maximises marketing campaign profitability while also explicitly incorporating demand uncertainty. A deterministic counterpart is also formulated to enable comparison.

The primary contribution of this study is the development of a recourse model that generates more reliable and adaptable decisions for practical retail environments. Although a customer-centric stochastic programming model is proposed, the primary focus of this study is to demonstrate the value and versatility of the stochastic programming approach. By applying it in the context of marketing campaign optimisation, the study aims to show how industrial engineers could leverage this methodology to enhance decision-making under uncertainty. While marketing serves as the illustrative case, the proposed approach applies in numerous industries and decision-making environments in which uncertainty plays a crucial role.

The remainder of the paper is structured as follows. First, a literature study of optimisation theory and the marketing industry is conducted in Section 2, followed by the development of the proposed mathematical model in Section 3. Next, Section 4 validates that a stochastic programming approach provides solutions that are hedged against uncertainty. Section 5 concludes the study, with Section 6 outlining recommendations for potential future work.

2. LITERATURE STUDY

Optimisation theory, with a specific focus on linear programming (LP), originated during the Second World War [11]. At that time, the military faced unique difficulties in allocating limited resources such as submarines, radars, and aircraft, and mathematics was used to solve these problems scientifically. LP has since attracted the attention of mathematicians, economists, numerical analysts, and decision scientists. Its goal is to derive the best possible solution for the defined decision variables.

Although many sources claim that LP was unknown before 1947, Dantzig [12] pointed out that its foundations existed earlier. For instance, Fourier wrote a paper on LP in 1823, and De la Vallée Poussin wrote one in 1911. Kantorovich also published a monograph in 1939, which was overlooked for ideological reasons [13].

Integer programming (IP) was introduced in 1958 by Gomory, although earlier work was done by Fulkerson and Dantzig. The branch-and-bound algorithm is one of the most effective methods for solving IP problems. IP has been used in investment, knapsack, fixed cost, and travelling salesman problems [14]. Bixby [15] and Anderson *et al*. [16] offer additional reading on IP, while Mallach [17], Stacho [18], and Vanderbei [19] explain standard ILP formulations.

Deterministic optimisation excludes uncertainty, assuming that all information is known in advance [6], or relies on estimated uncertainty based on historical data. Optimisation has been extensively applied in finance [20], portfolio selection [21], marketing [22], [23], and energy [24]. This study focuses on the application of optimisation, specifically mathematical programming, in the marketing industry.

Before the 1960s, marketing relied mainly on human judgement and intuition. During the early 1960s, optimisation models were introduced to support these decisions [25], [26]. Bisset and Terblanche [22] have drawn up a comprehensive timeline of marketing models:

- ***1960–1969*:** Marketing mix optimisation [27], including LP and goal programming (GP) [28], [29].
- ***1970–1979*:** Formulation of stochastic models [30], response models [31], and labelled decision models [32].
- ***1980–1989*:** Integration of machine learning and expert systems [33].
- ***1990–1999*:** Emergence of consumer choice and theoretical modelling concerning sales promotion [34], [35].
- ***2000–present*:** Customer-centric and churn prediction models, and customer lifetime value (CLV) approaches [36].

As technology progressed, so did marketing optimisation approaches. To address uncertainty, three main optimisation approaches are used: robust optimisation (RO), chance-constrained programming (CCP), and stochastic programming (SP) [37]. Models under uncertainty can be linear, integer, or mixed-integer. This paper focuses on SP, although brief overviews of RO and CCP are also included.

One of the first RO applications was by Soyster [38], followed by Ben-Tal and Nemirovski [39]. RO assumes that uncertainty is deterministic rather than probabilistic. It is less sensitive to data variations and focuses on feasible decisions for any future realisation [40]. RO enforces hard constraints without using probability distributions. Its primary advantage lies in computational efficiency [41]. The standard RO formulation is presented in Equations (1) and (2) [42].

Minimise

$$f_0(x) \tag{1}$$

Subject to

$$f_i(x, u_i) \leq 0, \forall u_i \in U_i = 1, \dots, m. \tag{2}$$

The decision variable is $x \in \mathbb{R}^n$, with uncertainty set $U_i \in \mathbb{R}^k$. Applications include capacity planning [43], manufacturing [44], and production scheduling [45]. For additional information, refer to Bertsimas [40] and Ben-Tal et al. [46].

CCP addresses the risk of constraint violation under uncertainty. Charnes and Cooper [20] gave the first deterministic equivalents under chance constraints. A typical CCP problem is provided in Equations (3) and (4) [47].

Minimise

$$f(c, x) \tag{3}$$

Subject to

$$P(Ax \leq b) \geq \alpha \tag{4}$$

Here, P represents the probability distribution, with α specifying the confidence level, where $0 \leq \alpha \leq 1$. The following reformulated expression results in Equation (5) [47]:

$$P(\sum_{j=1} a_{ij}x_j \leq b_i) \geq \alpha \tag{5}$$

Applications of CCP include supply chain [13] and power systems [48].

Dantzig [12] and Beale [49] introduced SP. According to Higle and Sen [8], SP is more complex than deterministic optimisation, as it applies random parameters and probability distributions over one or multiple periods.

A two-stage SP model segregates decisions into a first stage (before uncertainty is revealed) and a second stage (after uncertainty is realised) [50]. The first stage is based on historical or estimated data [22], while the second stage makes adjustments, based on outcomes. This adaptability is referred to as recourse [8], leading to the term “recourse model”. The standard formulation is presented in Equations (6) and (7).

Minimise

$$c^T x + E[Q(x, D)] \tag{6}$$

Subject to

$$x_0 \geq 0 \tag{7}$$

Multi-stage SP expands this logic in several stages. The standard formulation of a multi-stage SP problem is provided in Equation (8).

Minimise

$$c^T x_0 + E[Q_1(x_0, D_1)] \tag{8}$$

Unlike in two-stage SP models, $Q_1(x_0, D_1)$ is not explicitly determined.

A systematic literature review (SLR) performed by Bisset and Terblanche [22] investigated SP models. They found that 14% used single-stage models, whereas 43% used two-stage or multi-stage approaches. Some examples are:

- Single-stage SP for search advertising decisions [51].
- SP-based decision support for bid and pricing strategies [52].
- Two-stage SP for budgets, comparing stochastic and deterministic variants [53].

Of the nine studies reviewed, only two focused on marketing campaigns, with just one applying SP [52] in following a product-oriented rather than customer-oriented approach.

In the more recent literature, Sedlářová Nehézová et al. [54] provided a robust optimisation approach to budget allocation in online marketing campaigns. This model incorporates uncertainties in conversion costs by integrating fuzzy linguistic scales, allowing marketers to express uncertainty levels qualitatively. This transforms traditional deterministic models into more resilient frameworks, protecting against the cost coefficient fluctuations that are commonly encountered in digital marketing.

Hikima and Takeda [55] offered a novel pricing optimisation framework in which the uncertainty in demand is influenced by the pricing decisions themselves, challenging the traditional belief that uncertainty is independent of decision variables. The authors formulated a non-convex stochastic optimisation model and developed an unbiased stochastic gradient estimator with variance reduction to solve the problem efficiently. Using synthetic and real-world retail data, the method shows superior performance, yielding higher revenues than conventional methods. Thus, this approach gives a more realistic and effective solution for pricing under uncertainty, which is particularly relevant for revenue management in dynamic marketing scenarios.

In another study, Li and Yang [51] addressed the challenge of grouping keywords in sponsored search advertising campaigns under uncertainty. The authors proposed an SP model that considered click-through and conversion rates as random variables. Also, a branch-and-bound algorithm was developed to solve the model, and computational experiments showed its superiority over existing methods in respect of profitability and robustness.

Despite the growing body of research about stochastic programming in marketing-related applications, most studies focus on pricing strategies, keyword grouping, or budget allocation, often from a product-oriented perspective. The SLR by Bisset and Terblanche [22] revealed that, among the limited number of studies addressing marketing campaigns, only one applied a stochastic programming model, and it lacked a customer-centric approach. More recent studies, such as those by Sedlářová Nehézová et al. [54] and Hikima and Takeda [55], provided innovative models that address demand or cost uncertainty. However, these studies remain detached from the campaign-level, customer-targeting decisions that characterise strategic marketing interventions. In Section 3, this gap is addressed by proposing a distinct deterministic model that has been obtained from models already found in the literature.

3. MATHEMATICAL MODEL DEVELOPMENT

Section 3 presents the mathematical notation and formulations that characterise the proposed deterministic and recourse models. Section 3.1 defines the deterministic model, while Section 3.2 details the recourse model.

3.1. Model 1: Deterministic model

This section introduces a deterministic integer linear programming (ILP) model that has been developed to optimise marketing campaign decisions. The formulation of this deterministic model draws from established marketing optimisation models in the literature [5], [9], offering a combined decision-support model based on fixed input parameters. The model aims to determine which customers should be targeted with direct offers for a campaign, selecting the most appropriate marketing channel for each customer while identifying the optimal retail store in a specific period for product allocation to ensure maximum profitability.

The sets and indices of the model are defined as follows. Let $i \in I = \{1, \dots, |I|\}$ represent the set of customers and $j \in J = \{1, \dots, |J|\}$ the products pertaining to the campaign. Marketing channels are indexed by $k \in K = \{1, \dots, |K|\}$, retail stores by $l \in L = \{1, \dots, |L|\}$, and time periods by $t \in T = \{1, \dots, |T|\}$.

Two classes of costs are associated with campaign management: the marketing costs and the variable costs [5], [9]. The marketing cost C_{ijk} denotes the expense incurred when marketing product j is presented to customer i via channel k. The variable cost α_{ijlt} corresponds to the capital required to allocate product j to store l for customer i during period t. The return from a successful transaction is expressed by γ_{ijklt}, indicating the capital gained by the retailer if customer i purchased product j in store l via channel k during period t.

Two probabilities are considered in this deterministic model: the product probability and the inter-marketing time probability. The product probability β_{jlt} characterises the likelihood of product j being purchased in store l during period t independent of the customer or marketing channel. Retailers must estimate β_{jlt} before making any resource allocation decisions so as to avoid stores with low purchase probability. The inter-marketing time probability δ_{ilt} reflects the likelihood that customer i will purchase any project j in store l during period t, based on the historical delay between receiving an offer and making a purchase.

To illustrate, consider a scenario where customer 1 receives a direct offer for product 1. Based on historical campaign data, if 100 out of 1 000 customers bought product 1 from store 1, the estimated product probability is $100 \times 100/1000 = 10.00\%$. Further, if customer 1 typically purchases a product 10 days after receiving an offer, and the offer was sent 30 days before the campaign, then the inter-marketing probability is $100 \times 10/30 = 33.00\%$. Thus, multiplying these two probabilities, $\beta_{jlt} \times \delta_{ilt}$, presents a strong indication of the likelihood of this specific purchasing event. Although this study focuses on the model's formulation, these probabilities could be derived from historical transactional and consumer data.

Considering the preceding set definitions, additional model parameters are the following:

- M_i: Maximum number of offers that can be sent to customer i.
- m_j: Maximum number of allocations allowed for product j during the campaign.
- η_j: Fixed cost incurred when product j is used in the campaign.
- R: The corporate hurdle rate representing investment risk.
- B_j: Budget allocated to product j.
- N_k: Total number of marketing channels available, defined as the type of channel used, based on historical probability data.
- Q: Total number of promotional offers available in the campaign.
- T_{lt}: Indicator specifying whether store l is considered in period t.

The decision variable y_{ijklt} determines whether customer i receives an offer for product j, the marketing channel k used, the store l to which the product is allocated, and the specific period t. The complete deterministic ILP formulation is presented in Equations (9) to (17).

Maximise

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T}\left(\left((\gamma_{ijklt}-\alpha_{ijlt})\beta_{jlt}\delta_{ilt}\right)-C_{ijk}\right)y_{ijklt,} \tag{9}$$

Subject to

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T} y_{ijklt} \le Q, \tag{10}$$

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T}((\gamma_{ijklt}\ \beta_{jlt}\delta_{ilt})\,y_{ijklt}) \ge (1+R)\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T}(\alpha_{ijlt}+C_{ijk})y_{ijklt} \tag{11}$$

$$\sum_{i\in I}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T}(C_{ijk})y_{ijklt} \le B_j,\ \forall j\in J, \tag{12}$$

$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T} y_{ijklt} \le M_{i,}\ \forall i\in I, \tag{13}$$

$$\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T} y_{ijklt} \le N_k,\ \forall k\in K, \tag{14}$$

$$\sum_{i\in I}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T} y_{ijklt} \le m_{j,}\ \forall j\in J, \tag{15}$$

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} y_{ijklt} \le T_{lt,}\ \forall l\in L,\ \forall t\in T, \tag{16}$$

$$y_{ijklt}\in\{0,1\},\ \forall i\in I,\ \forall j\in J, \forall k\in K,\ \forall l\in L, \forall t\in T \tag{17}$$

The objective function in Equation (9) maximises the total profit generated from a marketing campaign. The variable cost is deducted from the return obtained from a specific transaction, which is multiplied by the two probabilities, indicating the likelihood of a specific purchasing event. Based on this profit, the marketing costs are deducted, which provides a profit estimate for maximisation.

To regulate the decisions of the model, several constraints are introduced. The constraint in Equation (10) limits the number of direct offers distributed throughout the campaign duration, thereby managing customer exposure. Equation (11) guarantees that the expected return from the campaign meets or exceeds the corporate hurdle rate R, thus aligning with organisational investment requirements. The budget constraint, presented in Equation (12), ensures that the total expenditure, including marketing and variable costs, does not exceed the predetermined budget of the campaign.

Additional constraints refine the operational feasibility of the model. Equation (13) describes whether a customer is eligible to receive a direct offer, whereas Equation (14) stipulates the number of marketing channels that may be used to prevent oversaturation and to manage complexity. Equation (15) provides a cumulative restriction on the sum of channels, customers, stores and time intervals per product. Last, Equation (16) ensures that product allocations to customers at specific retail stores and periods are made within practical and strategic limits. This constraint is also known as the maximum allocation (MA) constraint.

As explained in Section 2, deterministic models are limited by their inability to accommodate variability and uncertainty in real-world scenarios. Furthermore, a review of the literature reveals a gap in applying stochastic programming techniques to address the problem presented in this study. The next subsection introduces a stochastic extension of the deterministic model to address this shortcoming, incorporating uncertainty in customer behaviour, product uptake, and campaign dynamics.

3.2. Model 2: Recourse model

The proposed stochastic recourse model extends the proposed deterministic formulation that was obtained from various models in the literature [5], [9] by incorporating uncertainty, and is structured into two stages. All indices and parameters remain consistent with Model 1.

In the first stage, the model selects which customer i should receive an offer for project j via marketing channel k, using decision variable x, where $x = \{x_{ijk} : i \in I, j \in J, k \in K\}$ and marketing costs $C^T = \{-C_{ijk} : i \in I, j \in J, k \in K\}$. These decisions are executed prior to the campaign, and without knowing how customers will respond.

The second stage involves allocating the promoted products to retail stores once the customer behaviour becomes clearer. A recourse approach enables the model to adapt initial decisions under uncertainty. Three demand scenarios – low, medium, and high – are considered, where $S = \{low, medium, high\}$, each with a probability ρ_s. Scenario-specific parameters include the product probability β_{sjlt} and the inter-marketing time probability δ_{silt}, alongside variable costs α_{ijlt} and expected returns γ_{ijlt}.

Uncertainty is only introduced through β, where $\beta = \{\beta_{sjlt} : s \in S, j \in J, l \in L, t \in T\}$ and through ζ, where $\zeta = \{\zeta_{sijlt} : s \in S, i \in I, j \in J, l \in L, t \in T\}$. The second-stage decision variable ζ_{sijlt} indicates whether product j should be allocated to store l for customer i during period t under scenario s. The objective function for this two-stage stochastic programming model is provided in Equation (18).

Maximise

$$Q\big(x, (\beta_s, \delta_s)\big) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} P_{sijlt} \zeta_{sijlt}, \tag{18}$$

where

$$\zeta_{sijlt} \in \{0,1\}, \ \forall s \in S, \forall i \in I, \forall j \in J, \forall l \in L, \forall t \in T, \tag{19}$$

and

$$P_{sijlt} = \big(\gamma_{ijlt} - \alpha_{ijlt}\big)\beta_{sjlt}\delta_{silt}. \tag{20}$$

Equation (21) specifies the expected function formulation.

$$E[Q(x, (\beta,\delta))] = \sum_{s\in(l,m,h)} \rho_s \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}\Big((\gamma_{ijlt}-\alpha_{ijlt})\beta_{sjlt}\delta_{silt}\Big)\zeta_{sijlt}. \tag{21}$$

Therefore, Equations (22) to (33) provide the complete formulation of the recourse model.

Maximise

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}(-C_{ijk})x_{ijk} + \sum_{s\in(l,m,h)} \rho_s \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}((\gamma_{ijlt}-\alpha_{ijlt})\beta_{sjlt}\delta_{silt})\zeta_{sijlt} \tag{22}$$

Subject to

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} x_{ijk} \le L, \tag{23}$$

$$\sum_{s\in S}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T} \zeta_{sijlt} \le E, \tag{24}$$

$$\sum_{s\in S}\sum_{i\in I}\sum_{J\in J}\sum_{l\in L}\sum_{t\in T}((\gamma_{ijlt}\beta_{sjlt}\delta_{silt})\zeta_{sijlt}) \ge (1+R)\left(\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}(C_{ijk}x_{ijk}) + \sum_{s\in S}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}(\alpha_{ijlt}\zeta_{sijlt})\right), \tag{25}$$

$$\sum_{i\in I}\sum_{k\in K} C_{ijk}\, x_{ijk} \le B_{j,}\ \forall\in J, \tag{26}$$

$$\sum_{k\in K} x_{ijk} \ge \sum_{s\in S}\sum_{t\in T}\sum_{l\in L}\zeta_{sijlt},\ \forall i\in I, \forall j\in J, \tag{27}$$

$$\sum_{j\in J}\sum_{k\in K} x_{ijk} \le M_i,\ \forall i\in I, \tag{28}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le N_{k,}\ \forall k\in K, \tag{29}$$

$$\sum_{i\in I}\sum_{k\in K} x_{ijk} \le m_{j,}\ \forall j\in J, \tag{30}$$

$$\sum_{s\in S}\sum_{i\in I}\sum_{j\in J}\zeta_{sijlt} \le T_{lt,}\ \forall l\in L, \forall t\in T, \tag{31}$$

$$x_{ijk,} \in \{0,1\},\ \ \forall i\in I, \forall j\in J, \forall k\in K, \tag{32}$$

$$\zeta_{sijlt,} \in \{0,1\},\ \ \forall s\in S, \forall i\in I, \forall j\in J, \forall l\in L, \forall t\in T. \tag{33}$$

The objective function in Equation (22) maximises total campaign profitability by determining which customers should be targeted with offers and which marketing channels should be used in the first stage. It also specifies the optimal retail store allocation for each customer-product combination for maximised profitability. Akin to Equation (10) in Model 1, the maximum direct offer constraint in Equation (23) restricts the number of offers (L) issued in the first stage. In the second stage, the maximum product allocation (MPA) constraint in Equation (24) denotes the cumulative restriction (E) of store, product, and customer allocations summed across the complete timespan.

To guarantee investment viability, the corporate hurdle rate (CHR) constraint in Equation (25) enforces a minimum return of R. The budget constraint in Equation (26) assures that marketing costs remain within the financial limits of the specific campaign. The decision variable linkage constraint in Equation (27) prevents product allocation in the second stage unless a customer was targeted in the first stage, maintaining logical consistency between stages. This constraint is critical in distinguishing the model's two-stage structure.

Additional constraints guide offers and promotion decisions. Similar to Equation (13), the maximum customer quantity (MCQ) constraint in Equation (28) governs customer eligibility for offers. Corresponding with Equation (14), the maximum marketing channel quantity (MMCQ) constraint in Equation (29) regulates channel usage. Reflecting Equation (15), the maximum product quantity (MPQ) constraint in Equation (30) ensures that only selected products are promoted. Finally, the second-stage store allocation constraint in Equation (31) restricts the sum of all products and customer across stores for each time interval.

As discussed in Section 2, existing models do not address this problem under uncertainty. This proposed recourse model builds on established marketing principles, but is uniquely formulated to incorporate uncertainty and to deliver a robust model for improved decision-making.

4. DISCUSSION OF RESULTS

Section 4 discusses the results of the deterministic and recourse models using small-scale data to simplify the analysis and to demonstrate the benefits of a stochastic programming approach. While the models are not fully solved in this study, the focus is on showcasing the formulation of the recourse model and its value in managing uncertainty in decision-making. The insights gained are applicable in various industries to enhance decision-making.

Both models determine which customers to target, which marketing channels to use, and how to allocate products to retail stores within a campaign period. The key distinction is that the recourse model offers more robust solutions by accounting for uncertainty. Section 4.1 shows that the deterministic model fails to provide solutions that hedge against uncertainty, often leading to unreliable outcomes. In contrast, Section 4.2 illustrates how the recourse model effectively addresses this limitation, aligning with the study's objectives described in Section 1.

4.1. Deterministic approach to optimising a marketing campaign

This section illustrates that the proposed deterministic model fails to generate solutions that are hedged against uncertainty. It is worth noting that deterministic models base their decisions on a single assumed scenario – i.e., low, medium, high, or a mean value – before outcomes are known. To provide more context, Equation (34) introduces the objective function of the deterministic model, as formulated in Section 3.1.

Maximise

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T}(\left((\gamma_{ijklt}-\alpha_{ijlt})\beta_{jlt}\delta_{ilt}\right)-C_{ijk})y_{ijklt,} \tag{34}$$

The objective function in Equation (34) incorporates two key scenario-dependent parameters: the product purchase probability (β_{jlt}) and the average inter-marketing time probability (δ_{ilt}). These two probabilities significantly influence profitability, while the other parameters, such as costs, remain constant across scenarios. Thus, these two probabilities influence the profit and are linked to one specific scenario outcome $S = \{low, medium, high, MV\}$.

The set of scenarios is $S = \{low, medium, high\}$, and associated with each scenario $s \in S$ is a probability ρ_s. The mean value scenario is expressed as $S = \{MV\}$, and is also associated with a probability ρ_s. The probabilities are assumed and calculated for each scenario outcome, as shown in Table 1.

Table 1: Probability allocation per scenario

Scenario outcome	Probability (ρ_s)
Low	10%
Medium	70%
High	20%
Mean value (MV)	10% (low) + 70% (medium) + 20% (high)

Before issuing a direct offer, the retailer estimates the potential profit that a customer might deliver if a product is purchased during the campaign. This expected profit, classified as low, medium, or high, depends entirely on the scenario assumed in the deterministic model. While retailers aim to target customers who promise the highest returns, the likelihood of realising those high profits is often low because of uncertainty, as illustrated by the different probabilities in Table 1.

As Dyer *et al.* [4] noted, customer behaviour is influenced by numerous unpredictable factors, making it unlikely that outcomes will align perfectly with expectations. Since the deterministic model does not incorporate this uncertainty, retailers may instead prefer to target customers with a reasonable chance of purchasing and yielding a moderate profit. The latter would prevent the retailer from selecting a customer who might yield a high profit but who has a low probability, resulting in financial losses if the scenario does not occur.

Subsequently, the model assesses only one scenario from the set $S = \{low, medium, high, MV\}$ when making decisions. Both the product probability (β_{jlt}) and the average inter-marketing time probability (δ_{ilt}) are assumed to reflect the same scenario outcome – i.e., low probabilities under a low scenario and high probabilities under a high scenario – with a similar logic applying to the medium and mean value cases. To analyse this, a simplified dataset was used under the following assumptions:

- The dataset includes three customers, three products, three marketing channels, three retail stores, and three time periods.
- Expected returns γ_{ijklt} were fixed at R150.00, and variable costs α_{ijlt} at R40.00.
- Marketing costs C_{ijk} were randomly generated between R0.00 and R30.00, uniformly distributed and identical across scenarios.
- The probabilities β_{jlt} and δ_{ilt} were generated for each scenario: low ([0.00, 0.33]), medium ([0.33, 0.66]), and high ([0.66, 0.99]), all uniformly distributed.
- Product B_j was set at R50.00, and the return on investment (ROI) threshold at 30.00%.
- A maximum of one direct offer could be issued, with reference to Equation (22).
- No additional restrictions were applied beyond the standard model constraints.

The scenario probabilities were assigned as follows: 10% (low), 70% (medium), 20% (high). The mean value (MV) scenario represents the weighted average of the scenarios. The profit per scenario is shown in Table 2.

Table 2: Profit per scenario and the mean value

Profit (Rands)			
The mean value scenario	The three individual scenario outcomes		
	Low (10%)	Medium (70%)	High (20%)
37.74	0.00	36.52	89.57

Table 2 summarises the profits in each case: R0.00 (low – 10%), R36.52 (medium – 70%), R89.57 (high – 20%), and R37.74 (mean value). Although the high scenario yields the highest profit of R89.57, it is expected to occur with a probability of only 20%. Planning based on this scenario would be risky. In contrast, the medium scenario results in a profit of R36.52 with a probability of 70%, which is a more reasonably safe scenario to assume, even if the profit is less than that for the high scenario. The low scenario results in no profit, as the ROI threshold is not met owing to low probability values. The mean value scenario provides a more balanced outcome, but still fails to account for the full range of possible scenarios.

Since only one scenario is assumed and planned for accordingly, these results underline a critical limitation: deterministic models cannot accommodate uncertainty across different outcomes. These models assume perfect foresight, and do not adapt when actual conditions deviate from the assumed scenario. As highlighted by Higle and Sen [8], such models often result either in missed opportunities when a medium scenario is assumed or in financial loss when a high scenario is assumed. Consequently, relying solely on deterministic solutions is not advisable for robust decision-making. A stochastic approach is needed to hedge decisions better against the variability inherent in real-world environments.

4.2. Stochastic approach to optimising a marketing campaign

The purpose of this section is to show that the proposed recourse model produces effective solutions that are hedged against uncertainty. The method applied in this section is based on the work of Higle and Sen [8]. Contrary to the deterministic model that optimises on the basis of a single assumed scenario, the recourse model investigates decisions in all three possible outcomes simultaneously – i.e., $S = \{low, medium, high\}$. To show this, Equation (35) provides the objective function of the recourse model, as formulated in Section 3.2.

Maximise

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\left(-C_{ijk}\right)x_{ijk} + E[Q(x,\ (\beta,\delta))], \tag{35}$$

where

$$E[Q(x,\ (\beta,\delta))] = \sum_{s\in(l,m,h)}\rho_s\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}\left(\left(\gamma_{ijlt}-\alpha_{ijlt}\right)\beta_{sjlt}\delta_{silt}\right)\zeta_{sijlt}, \tag{36}$$

And

$$P_{sijlt} = \left(\gamma_{ijlt}-\alpha_{ijlt}\right)\beta_{sjlt}\delta_{silt}. \tag{37}$$

As in the deterministic model, profit depends exclusively on the product probability β_{sjlt} and the average inter-marketing time probability δ_{silt}, both scenarios being specific. The same baseline data and structure from Section 4.1 are applied, with a few key differences:

- Marketing costs C_{ijk} are scenario-dependent and are drawn from [0,10] for low, [10,20] for medium, and [20,30] for high scenarios. A mean value is also computed.
- The probabilities β_{jlt} and δ_{ilt} are randomly generated for each scenario range: low ([0.00–0.33]), medium ([0.33–0.66]), and high ([0.66–0.99]), with uniform distribution.
- The maximum direct offer constraint is set to 1 (only one strategy may be selected).
- The maximum allocation constraint in Equation (16) is set so that the model selects a strategy for period 2 only, allowing focused analysis.
- No additional constraints are imposed beyond these adjustments.

In contrast to the deterministic model, the recourse model examines the full range of potential outcomes, resulting in more reliable and risk-aware decisions. This section aims to highlight how such a model better supports campaign planning under uncertainty.

The deterministic model was solved separately for each scenario $S = \{low, medium, high, MV\}$ while the recourse model was solved once by considering all three scenarios simultaneously - i.e., $S = \{low, medium, high\}$. As a result, the deterministic model produced four separate profit values, whereas the recourse model yielded a single profit value. These outcomes are summarised in Table 3.

Table 3: Profit involving deterministic and recourse models

Profit (Rands)				
Deterministic model: Mean value	**Deterministic model: Per scenario**			**Recourse model**
	Low (10%)	**Medium (70%)**	**High (20%)**	
28.80	0.00	31.55	79.90	37.73

Table 3 shows that deterministic model profits vary widely, depending on the assumed scenario (low, medium, high, or mean value), ranging from R0.00 to R79.90. However, each of these values is based on a single assumed outcome, as explained in Section 4.1. The recourse model provides only a single profit, as it simultaneously investigates decisions for all three possible outcomes.

To highlight how the recourse model hedges against uncertainty, each deterministic scenario was also evaluated using the recourse model's structure [8]. Specifically, the same marketing costs C_{ijk} used in the deterministic model were applied in the first stage of the recourse model. The second-stage values for the product probability β_{jlt} and the inter-marketing time probability δ_{ilt} were generated using the same three-scenario setup for all cases. This allowed for a direct comparison between the assumed outcome from the deterministic model and the expected profit when uncertainty was considered. Thus, Table 4 illustrates what would have happened if a retailer had assumed a specific scenario, as required in a deterministic approach, and then shows how the actual outcome could have differed when uncertainty was accounted for in the second stage, demonstrating that the initial assumption may not hold in practice.

Table 4: Comparison of different profits

Scenario outcome	Recourse model profit (Rands)	Deterministic model profits (Rands)	Expected profits (Rands)
Low (10%)	37.73	0.00	43.60
Medium (70%)		31.55	35.27
High (20%)		79.90	22.49
Mean value	N/A	28.80	33.03

Table 4 shows that the recourse model's profit of R37.73 is consistently closer to the expected profits for all scenarios than the deterministic model's results. For example, while the deterministic model projected a profit of R79.90 under the high scenario with a 20% probability, the actual expected profit once uncertainty was introduced was only R22.49. This reflects a significant overestimation and a potential financial risk if the decision-maker assumed a high scenario. In contrast, while the deterministic model projected a profit of R31.55 under the medium scenario with a 70% probability, the actual expected profit once uncertainty was introduced was R35.27. This reflects a significant underestimation and a potential financial gain if the decision-maker assumed a medium scenario. This analysis illustrates that deterministic models, when used independently, may lead to suboptimal decisions, whereas the recourse model offers more reliable, uncertainty-aware solutions that are consistent with real-world variability. By incorporating scenario variability, the recourse model offers a more balanced and realistic view of profit outcomes.

To illustrate how uncertainty influences profitability, percentage differences were calculated between the profits generated by each model and the expected profits obtained from the recourse model. These results are summarised in Table 5.

Table 5: Percentage difference between recourse model and expected profit

Scenario outcome	Recourse model profit (Rands)	Expected profits (Rands)	Percentage difference
Low (10%)	37.73	43.60	14.44%
Medium (70%)		35.27	6.74%
High (20%)		22.49	50.61%
Mean value	N/A	33.03	13.28%

These results show that the recourse model consistently delivers profits close to the expected values for each scenario. Even in the high scenario, where the difference is 50.61%, the model still provides a reasonable result, enabling the retailer to recover without incurring significant loss if such a scenario were to occur. For the low scenario, the smaller gap of 14.44% depicts minimal missed opportunity. Overall, this confirms that the recourse model offers a balanced and risk-aware solution, regardless of which scenario occurs. Table 6 shows the same comparison for the deterministic model.

Table 6: Percentage difference between deterministic model and expected profit

Scenario outcome	Deterministic model profits (Rands)	Expected profits (Rands)	Percentage difference
Low (10%)	0.00	43.60	200.00%
Medium (70%)	31.55	35.27	11.13%
High (20%)	79.90	22.49	112.14%
Mean value	28.80	33.03	13.68%

The deterministic model shows large deviations in the low and the high scenarios. For example, in the high scenario, an overestimation of 112.14% would lead to a significant misallocation of resources or financial losses. Similarly, assuming a low scenario leads to a complete loss (R0.00 profit), missing substantial opportunities. These discrepancies illustrate that planning based on a single assumed scenario is risky and unreliable. A direct comparison between the two models is presented in Table 7:

Table 7: Percentage difference between recourse model and deterministic model

Scenario outcome	Recourse model profits	Deterministic model profit
Low (10%)	14.44%	200.00%
Medium (70%)	6.74%	11.13%
High (20%)	50.61%	112.14%
Mean value	13.28%	13.68%

Table 7 illustrates that the recourse model outperforms the deterministic model in managing variability in all scenarios. Although the recourse model does not perfectly align with the expected values, it substantially reduces the margin of error. This is expected, as first-stage decisions must accommodate multiple future outcomes. Thus, the strength of the recourse model lies in its flexibility, as it minimises the risk if the assumed scenario does not materialise while limiting missed opportunities. In contrast, the deterministic model lacks this adaptability. Deterministic models make decisions on the basis of a single scenario, which is unlikely to hold in practice, rendering it an unreliable strategy under uncertainty. Thus, to support more robust decision-making, a model is required that balances outcomes across all plausible futures.

The recourse model satisfies this need by simultaneously considering all three scenarios during optimisation. As illustrated by the results, it offers hedged, stable solutions, and proves that a stochastic programming approach is far more secure and dependable when managing demand uncertainty.

5. SUMMARY AND CONCLUSION

Retailers today face increasing difficulty in managing demand uncertainty, particularly when trying to maximise the profitability of marketing campaigns. Despite extensive research, the literature lacks a two-stage stochastic programming model that addresses this difficulty. This study fills the gap by proposing a unique two-stage stochastic model that maximises campaign profitability and provides robust solutions to uncertainty. The study has outlined the fundamentals of campaign management, followed by a review of the recent mathematical developments in marketing optimisation. A new deterministic model, built on the marketing principles of two existing models, has been introduced to determine which customers to target, the most appropriate marketing channels, and optimal product allocation across stores and periods.

Building on this, the study's primary contribution is the development of a two-stage recourse model. Unlike the deterministic approach, the recourse model incorporates multiple possible future outcomes, allowing decisions to be hedged against uncertainty. The results clearly show the added value of a stochastic programming approach in improving decision-making under uncertainty. In conclusion, this research introduces both a practical and a theoretical advancement, offering a more reliable framework for retailers who seek to optimise campaign performance in uncertain environments.

6. FUTURE RESEARCH

The results confirm that the proposed recourse model maximises campaign profitability and produces solutions that are hedged against uncertainty. This study intentionally used small-scale data to simplify the analysis and to enable a focused validation of the model's behaviour. However, the model's true potential lies in its application in larger and more realistic problem instances. In real-world settings, campaigns involve numerous products, customers, and periods. As these dimensions increase, so does the computational complexity. The decision variables concerning both stages of the recourse model are integer-valued, making the problem non-deterministic polynomial-time (NP) hard, especially when multiple scenarios are considered. Therefore, to address this, future research could explore advanced solution algorithms, such as the L-shaped method, which approximates continuous distributions and improves solution quality by iteratively refining upper and lower bounds. As noted in Section 2, other decomposition-based methods could exploit the model's structure and enhance its scalability. Using these techniques could extend the model to realistic campaign environments, providing accurate and timely solutions despite increased problem size.

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