STOCHASTIC MODEL FOR COMMON CAUSE FAILURES AND HUMAN ERROR

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ABSTRACT

A consistent asymptotic normal (CAN) estimator and confidence limits for the steady-state availability of series and parallel systems subject to unit failures, common-cause shock (CCS) failures and human error are studied. This paper also deals with the estimation from a Bayesian viewpoint with a number of prior distributions assumed for the unknown parameters in the system, which reflect different degrees of belief on the failure mechanisms. A Monte Carlo simulation is used to derive the posterior distribution for the steady-state availability and subsequently the highest posterior density (HPD) intervals. A numerical example illustrates the results.

OPSOMMING

'n Konsekwente asimptotiese normaalberamer en vertroueintervalle vir die ewewigstoestandsbeskikbaarheid van stelsels in serie en parallel, wat onderworpe is aan eenheids-, gemeenskaplike skok- en menslike foutfalings, word bestudeer. In die artikel word ook 'n Bayes-benadering gevolg vir die beraming deur 'n aantal a priori-verdelings vir die onbekende parameters in die stelsel, wat verskillende grade van vertroue in die falingsmeganismes weerspieël, te aanvaar. Monte Carlo-simulasie word gebruik om die a posteriori-verdeling vir die ewewigstoestandsbeskikbaarheid en daarna die hoogste a posteriori-digtheidsintervalle (HPD) af te lei. 'n Numeriese voorbeeld illustreer die resultate.

1. INTRODUCTION

Vesely [10] referred to the Reactor Safety study (WASH-1400 [9]) that states that the treatment of CCS failures is extremely important in assessments of the risk associated with nuclear power plant accidents. The CCS failure could be attributed to internal factors like design deficiencies, fabrication, etc. and external ones like environmental conditions (temperature, dust, humidity), power failure, fire, flood, earthquake, etc. The concept of human error according to Dhillon and Rayapati [3] is defined as a failure to perform a prescribed task (or the performance of a prohibited action), which could result in damage to equipment and property or disruption of scheduled operations. According to the work by Meister [7] about 30% of failures are directly or indirectly due to human errors. Some of the causes of human errors are wrong actions, maintenance errors, misinterpretation of instruments, etc. In order to predict realistic reliability and availability of systems, the occurrence of CCS failures and human errors must be considered [4].

El-Damcese [5] presented a model representing two unit multiplex systems with CCS failure and human error. In this paper a CAN estimator and confidence limits for the performance measure, steady-state availability, are derived for this two unit system, as well as the HPD intervals using a number of prior distributions. The advantages of the Bayes approach are that prior information and/or technical knowledge of the system can be incorporated into the inferential procedure and interval estimates can be obtained without relying on asymptotic results; hence are accurate even for small and moderate samples. It is assumed that prior knowledge is available about the parameters of the model, based on past experience with similar reliability data and that this prior knowledge can be mathematically translated into suitable prior densities as in section 4. With little or no prior information about a parameter, a non-informative prior distribution can be used to represent the knowledge of the parameter, this is also discussed in section 4.

Section 2 presents the necessary notation, description of the model and the expressions for the steady-state availability of the two different configurations. The confidence limits for steady-state availability are studied in section 3 and the Bayesian approach to this problem is introduced in section 4. The results are illustrated numerically in the last section.

2. SYSTEM DESCRIPTION AND NOTATION

The following assumptions are associated with the model:

(i) The system consists of two identical units. The failure rate and the repair rate are constant with parameters λ and μ respectively.

(ii) The system consists of CCS failures which are S-independent. The failure rate and repair rate of CCS failures are constant with parameters λ_c and μ_c respectively.

(iii) If the system fails due to human error, the failure rate and repair rate in that case are assumed to be exponential with parameters λ_h and μ_h respectively.

(iv) The unit failure, CCS failure and human error are assumed to be independent.

Notation

$P_i(t)$	Probability that the system is in state i (i = 0, 1, 2) at time t
$A_S(\infty)$	Steady-state availability of the series configuration in the presence of CCS failures and human error.
$A_P(\infty)$	Steady-state availability of the parallel configuration in the presence of CCS failures and human error.

The expressions for steady-state availability of the two configurations are as follows (see El-Damcese [5]):

$$A_S(\infty) = P_0 \tag{1}$$

$$A_P(\infty) = \sum_{j=0}^{4} P_j \tag{2}$$

$$P_0 = \frac{1}{1+\sum_{i=1}^{9} a_i} \quad \text{and} \quad P_j = a_j P_0, \text{ for } j = 1, 2, 3, ..., 9$$

where

$$a_1 = a_2 = \rho,\ a_3 = a_4 = \rho_h,\ a_5 = \rho^2,\quad a_6 = a_7 = \rho_h \lambda + \rho \lambda_h,\ a_8 = \rho_h^2,\ a_9 = \rho_c$$

and

$$\rho = \frac{\lambda}{\mu},\ \rho_h = \frac{\lambda_h}{\mu_h},\ \rho_c = \frac{\lambda_c}{\mu_c} \quad (\rho\text{'s are the utilization factors}).$$

3. CONFIDENCE LIMITS FOR STEADY-STATE AVAILABILITY

Let $\left(U_{11}, U_{12}, ..., U_{1n_1}\right)$, $\left(U_{21}, U_{22}, ..., U_{2n_2}\right)$ and $\left(U_{31}, U_{32}, ..., U_{3n_3}\right)$ be random samples of unit failures, CCS failures and failures due to human error with sample sizes n_1, n_2 and n_3 respectively. Let $\left(U_{41}, U_{42}, ..., U_{4n_4}\right)$, $\left(U_{51}, U_{52}, ..., U_{5n_5}\right)$ and $\left(U_{61}, U_{62}, ..., U_{6n_6}\right)$ be random samples of repairs of a unit, of the system due to a CCS failure and failure due to human error with sample sizes n_4, n_5 and n_6 respectively. All these samples are drawn from exponential populations. In particular we assume that $n_i = n;\ i=1, 2, \ldots, 6$.

Let $\theta_1 = \frac{1}{\lambda}$, $\theta_2 = \frac{1}{\lambda_c}$, $\theta_3 = \frac{1}{\lambda_h}$, $\theta_4 = \frac{1}{\mu}$, $\theta_5 = \frac{1}{\mu_c}$ and $\theta_6 = \frac{1}{\mu_h}$. The maximum likelihood estimator (MLE) of θ_i is given by $\overline{U}_i = \frac{1}{n}\sum_{j=1}^{n} U_{ij}$, $i = 1,2,...,6$. Hence from (1)

$$\hat{A}_S(\infty) = k^{-1}\overline{U}_1^2\overline{U}_2\overline{U}_3^2$$

and from (2)

$$\hat{A}_P(\infty) = k^{-1}\overline{U}_1\overline{U}_2\overline{U}_3\left(\overline{U}_1\overline{U}_3 + 2\overline{U}_4\overline{U}_3 + 2\overline{U}_6\overline{U}_1\right)$$

where

$$k = \overline{U}_1^2\overline{U}_2\overline{U}_3^2 + 2\overline{U}_1\overline{U}_2\overline{U}_3^2\overline{U}_4 + 2\overline{U}_1^2\overline{U}_2\overline{U}_3\overline{U}_6 + \overline{U}_4^2\overline{U}_3^2\overline{U}_2 + 2\overline{U}_1\overline{U}_2\overline{U}_3\overline{U}_6$$

$$+ 2\overline{U}_1\overline{U}_2\overline{U}_3\overline{U}_4 + \overline{U}_6^2\overline{U}_1^2\overline{U}_2 + \overline{U}_5\overline{U}_1^2\overline{U}_3^2.$$

By an application of the Multivariate Central Limit Theorem [8], it follows that

$$\sqrt{n}\left(\overline{X} - \theta\right) \underset{\rightarrow}{d} N_6(0, \Sigma)$$

where

$$\overline{X} = \left(\overline{U}_1, \overline{U}_2, \overline{U}_3, \overline{U}_4, \overline{U}_5, \overline{U}_6\right) \text{ and } \theta = \left(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\right)$$

The dispersion matrix $\sum = \left[\sigma_{ij}\right]_{6\times 6}$ is given by

$$\Sigma = \text{diag}\left(\theta_1^2, \theta_2^2, \theta_3^2, \theta_4^2, \theta_5^2, \theta_6^2\right)$$

From Rao[8], for $n \to \infty$,

$$\sqrt{n}\left(\hat{A}_j(\infty) - A_j(\infty)\right) \underset{\rightarrow}{d} N\left(0, \sigma_j^2(\theta)\right)$$

with

$$\sigma_j^2(\theta) = \sum_{i=1}^{6}\left(\frac{\partial A_j(\infty)}{\partial \theta_i}\right)^2 \sigma_{ii} = \sum_{i=1}^{6}\left(\frac{\partial A_j(\infty)}{\partial \theta_i}\right)^2 \theta_i^2 \quad \text{for } j = S, P.$$

Now, let $\hat{\sigma}_j^2 = \sigma_j^2\left(\hat{\theta}\right)$ be the estimator of $\sigma_j^2(\theta)$, obtained by replacing θ by its consistent estimator

$$\hat{\theta} = \left(\overline{U}_1, \overline{U}_2, \overline{U}_3, \overline{U}_4, \overline{U}_5, \overline{U}_6\right).$$

Since $\sigma_j^2(\theta)$ is a continuous function of θ, $\hat{\sigma}_j^2$ is a consistent estimator of $\sigma_j^2(\theta)$, i.e.

$$\hat{\sigma}_j^2 \to \sigma_j^2(\theta) \text{as } n \to \infty.$$

By Slutzky's theorem

$$\frac{\sqrt{n}\left(\hat{A}_j(\infty)-A_j(\infty)\right)}{\hat{\sigma}_j}\underset{\rightarrow}{d}\,N(0,1)\text{ as } n\rightarrow\infty.$$

Hence, the 100(1 - α)% asymptotic confidence limits for $A_j(\infty)$ are given by

$$\hat{A}_j(\infty)\pm k_{\frac{\alpha}{2}}\frac{\hat{\sigma}_j}{\sqrt{n}}\text{ for } j=S,P.$$

($k_{\frac{\alpha}{2}}$ is obtained from the normal tables)

4. BAYESIAN ANALYSIS OF STEADY-STATE AVAILABILITY

The likelihood function is given by

$$L(\lambda,\lambda_c,\lambda_h,\mu,\mu_c,\mu_h|T_1,T_2,T_3,T_4,T_5,T_6)$$

$$=(\lambda\lambda_c\lambda_h\mu\mu_c\mu_h)^n e^{-(\lambda T_1+\lambda_c T_2+\lambda_h T_3+\mu T_4+\mu_c T_5+\mu_h T_6)} \tag{3}$$

where $\left(T_i=\sum_{j=1}^{n}U_{ij}\quad i=1,2,...,6\right)$ is sufficient for $(\lambda,\lambda_c,\lambda_h,\mu,\mu_c,\mu_h)$.

4.1 Two parameter gamma density

If the analyst possesses more detailed information on λ, for example in terms of the mean value ω_1 and a standard deviation σ_1, he can formalize his prior information through a gamma prior distribution (denoted by $G(\nu_1;\gamma_1)$) for λ with probability density function (p.d.f.)

$$g(\lambda)=\frac{1}{\Gamma(\nu_1)}\gamma_1^{\nu_1}e^{-\gamma_1\lambda}\lambda^{\nu_1-1};\quad \lambda,\gamma_1,\nu_1>0. \tag{4}$$

The prior mean ω_1 and variance σ_1^2 are given by $\omega_1=\frac{\nu_1}{\gamma_1}$ and $\sigma_1^2=\frac{\nu_1}{\gamma_1^2}$, so that the prior information can be easily converted into suitable values of the prior parameters (hyperparameters)

$$\nu_1=\frac{\omega_1^2}{\sigma_1^2},\quad \gamma_1=\frac{\omega_1}{\sigma_1^2}. \tag{5}$$

Similary, assume $G(\nu_i;\gamma_i)$, $i=2,3,...,6$ as prior distributions for $\lambda_c,\lambda_h,\mu,\mu_c$ and μ_h respectively. Assume independence among the parameters $\lambda,\lambda_c,\lambda_h,\mu,\mu_c$ and μ_h. The flexibility present in the gamma family through the choices of the hyperparameters allows the analyst to select the model that best expresses the current state of knowledge about the parameter.

The joint posterior distribution, according to Bayes' theorem (using (3) and (4)) is defined by

$$g(\lambda, \lambda_c, \lambda_h, \mu, \mu_c, \mu_h | data) \propto \lambda^{n+\nu_1-1} \lambda_c^{n+\nu_2-1} \lambda_h^{n+\nu_3-1} \mu^{n+\nu_4-1} \mu_c^{n+\nu_5-1} \mu_h^{n+\nu_6-1} \cdot e^{-(\lambda(T_1+\gamma_1)+\lambda_c(T_2+\gamma_2)+\lambda_h(T_3+\gamma_3)+\mu(T_4+\gamma_4)+\mu_c(T_5+\gamma_5)+\mu_h(T_6+\gamma_6))} \tag{6}$$

Remark

1. The Jeffreys' prior for $(\lambda, \lambda_c, \lambda_h, \mu, \mu_c, \mu_h)$ [2] is given by

$$g(\lambda, \lambda_c, \lambda_h, \mu, \mu_c, \mu_h) \propto \frac{1}{(\lambda \lambda_c \lambda_h \mu, \mu_c \mu_h)}; \quad \lambda, \lambda_c, \lambda_h, \mu, \mu_c, \mu_h > 0 \tag{7}$$

with the joint posterior distribution as

$$g(\lambda, \lambda_c, \lambda_h, \mu, \mu_c, \mu_h | data) \propto (\lambda \lambda_c \lambda_h \mu \mu_c \mu_h)^{n-1} e^{-(\lambda T_1 + \lambda_c T_2 + \lambda_h T_3 + \mu T_4 + \mu_c T_5 + \mu_h T_6)} \tag{8}$$

If $\nu_i = \gamma_i = 0 \ (i = 1, 2, ..., 6)$ then (6) simplifies to (8).

2. If the standard gamma prior with p.d.f.

$$g(\lambda) = \frac{1}{\Gamma(\nu_1)} e^{-\lambda} \lambda^{\nu_1 - 1}; \quad \lambda, \nu_1 > 0 \tag{9}$$

is assumed for λ, the joint posterior distribution follows from (6) with $\gamma_i = 1 (i = 1, 2, ..., 6)$.

From the joint posterior distribution (6), $A_S(\infty)$ and $A_P(\infty)$ are obtained using Monte Carlo simulation methods, since each parameter is unconditionally gamma distributed. Values are generated from these 3 gamma distributions and substituted in (1) and (2) respectively- the expressions for the steady-state availability of the two configurations. Subsequently the posterior distribution are simulated for the steady-state availability for series and parallel systems, from which the HPD intervals can be obtained.

4.2 Beta distribution

If the prior distribution is chosen to be a beta distribution of the second kind (beta-prime or inverted-beta-2; denoted by $BP(m_1;r_1)$) for λ with p.d.f.

$$g(\lambda) = \frac{1}{B(m_1, r_1)} \frac{\lambda^{m_1 - 1}}{(1+\lambda)^{m_1 + r_1}}; \ \lambda, m_1, r_1 > 0 \tag{10}$$

with suitable values of the hyperparameters as

$$m_1 = \frac{\omega_1\left(\omega_1 + \omega_1^2 + \sigma_1^2\right)}{\sigma_1^2}, \quad r_1 = \frac{\omega_1\left(\omega_1 + \omega_1^2 + 2\sigma_1^2\right)}{\sigma_1^2} \tag{11}$$

where ω_1 and σ_1^2 are the prior mean and variance respectively. Similarly, assume $BP(m_i; r_i)$, $i = 2, \ldots, 6$, as prior distributions for $\lambda_c, \lambda_h, \mu, \mu_c$ and μ_h respectively. Assume independence among the parameters $\lambda, \lambda_c, \lambda_h, \mu, \mu_c$ and μ_h. The joint posterior distribution is given by

$$g(\lambda,\ \lambda_c, \lambda_h, \mu, \mu_c, \mu_h | data) \propto \frac{\lambda^{n+m_1-1}\lambda_c^{n+m_2-1}\lambda_h^{n+m_3-1}\mu^{n+m_4-1}\mu_c^{n+m_5-1}\mu_h^{n+m_6-1}}{(1+\lambda)^{m_1+r_1}(1+\lambda_c)^{m_2+r_2}(1+\lambda_h)^{m_3+r_3}(1+\mu)^{m_4+r_4}} \cdot \frac{e^{-(\lambda T_1 + \lambda_c T_2 + \lambda_h T_3 + \mu T_4 + \mu_c T_5 + \mu_h T_6)}}{(1+\mu_c)^{m_5+r_5}(1+\mu_h)^{m_6+r_6}}. \tag{12}$$

From (12), follows that each parameter is generalized gamma distributed as defined by Agarwal and Kalla [1] with the normalizing constant that can be represented in terms of the confluent hypergeometric function of the second kind (or the generalized gamma function defined by Kobayshi [6]). Monte Carlo simulation-methods can be used again, or alternatively Gibbs sampling to obtain the posterior distribution for the steady-state availability for series and parallel systems, from which the HPD intervals follows.

5. SIMULATION STUDY AND COMPARISIONS

To illustrate the results in sections 3 and 4, different exponentially distributed samples were simulated for the six variables in the system. The sample information is given in Appendix A. The following specific selection of parameter values:
$\lambda = 0.1$; $\lambda_c = \lambda_h = 0.07$; $\mu = \mu_c = \mu_h = 3$ were used in the illustration, except in Table 3. Since it is a simulation study the true values for the steady-state availability for the two configurations are known ($A_S(\infty) = 0.87134$; $A_P(\infty) = 0.97009$, except in Table 3). Table 1 shows the CAN estimator and confidence intervals (C.I.) for $A_S(\infty)$ and $A_P(\infty)$.

Initially 10 000 values are simulated from each distribution of the 6 parameters and subsequently the distribution of the steady-state availability is obtained. Figure 1 shows the 99% HPD intervals and posterior means (PM) for $A_S(\infty)$ for increasing sample sizes (assuming Jeffreys' prior). The variation can most probably be ascribed to the effect of sampling (also see Table 2).

In tables 2, 3, 4a, 4b and 5, the Bayesian estimation study of the steady-state availability for both parallel and series system is presented for different sample sizes. It is evident that the intervals are much smaller for the larger sample sizes.

	$A_S(\infty)$			$A_P(\infty)$		
n	**MLE**	**95% C.I.**	**99% C.I.**	**MLE**	**95% C.I.**	**99% C.I.**
40	0.8311	(0.7891; 0.8731)	(0.7759; 0.8863)	0.9648	(0.9544; 0.9751)	(0.9512; 0.9784)
100	0.8497	(0.8253; 0.8741)	(0.8176; 0.8818)	0.9694	(0.9635; 0.9752)	(0.9617; 0.9770)
500	0.8631	(0.8534; 0.8728)	(0.8504; 0.8759)	0.9701	(0.9675; 0.9727)	(0.9667; 0.9735)
700	0.8648	(0.8567; 0.8729)	(0.8542; 0.8754)	0.9705	(0.9683; 0.9726)	(0.9676; 0.9733)
1000	0.8692	(0.8627; 0.8757)	(0.8606; 0.8778)	0.9705	(0.9686; 0.9723)	(0.9680; 0.9729)

Table 1: CAN estimator and C.I. for $A_S(\infty)$ and $A_P(\infty)$

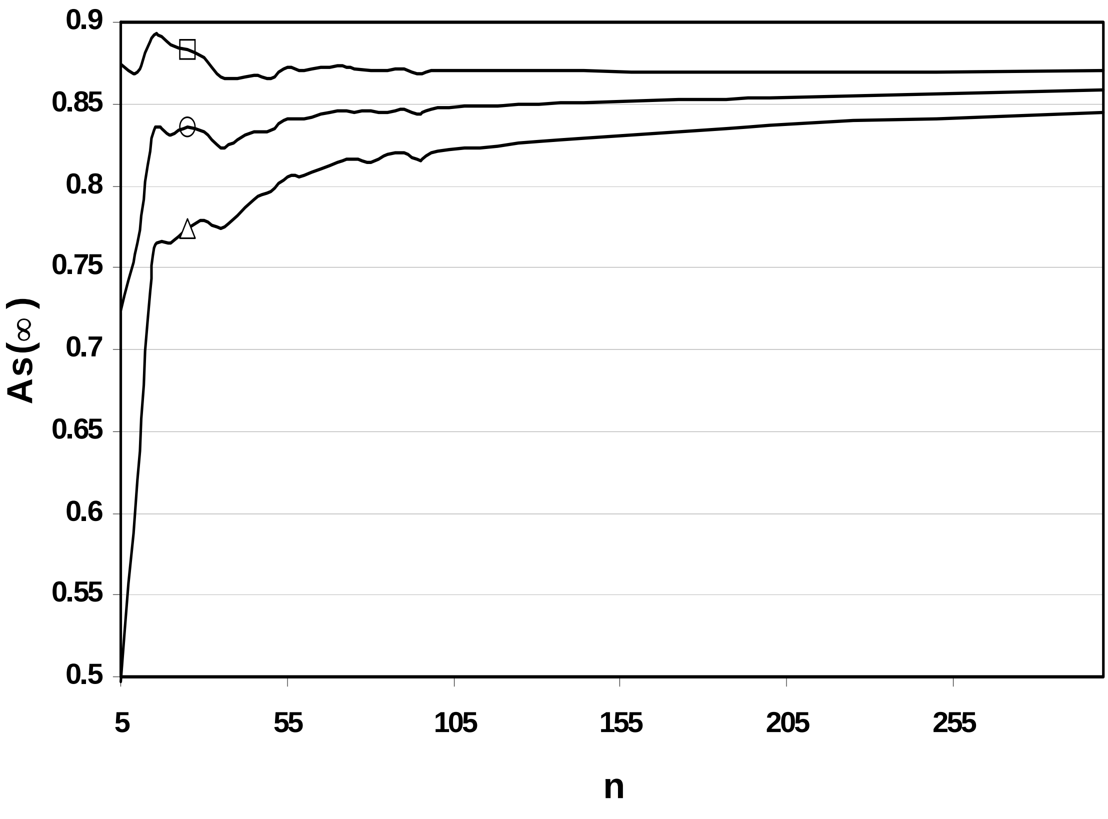

Figure 1: 99% HPD interval and PM for $A_S(\infty)$ for different sample sizes
o : PM □ : upper limit Δ : lower limit

Finally, the different priors are examined. Tables 4a, 4b and 5 show the posterior means and 95% HPD intervals for $A_S(\infty)$ and $A_P(\infty)$ respectively, under the prior distributions (i) Jeffreys' prior (see (7)), (ii) standard gamma prior density (see (9)), (iii) two parameter gamma prior density (see (4)) and (iv) beta prior (see (10)). The results are compared for gamma and beta priors. Different values for the hyperparameters of λ_h are used to illustrate different prior information. The influence on the posterior mean and the intervals is clear from the tables.

	$A_S(\infty)$				$A_P(\infty)$			
n	**PM**	**st.dev.**	**95% HPD**	**99% HPD**	**PM**	**st.dev.**	**95% HPD**	**99% HPD**
10	0.7604	0.0601	(0.6292; 0.8608)	(0.5988; 0.8807)	0.9538	0.0148	(0.9203; 0.9768)	(0.9096; 0.9811)
20	0.8301	0.0290	(0.7699; 0.8819)	(0.7501; 0.8912)	0.9635	0.0076	(0.9461; 0.9773)	(0.9409; 0.9805)
40	0.8293	0.0209	(0.7862; 0.8677)	(0.7756; 0.8817)	0.9644	0.0055	(0.9523; 0.9739)	(0.9472; 0.9764)
100	0.8474	0.0128	(0.8211; 0.8714)	(0.8093; 0.8791)	0.9688	0.0031	(0.9625; 0.9745)	(0.9601; 0.9766)
500	0.8628	0.0052	(0.8519; 0.8727)	(0.8470; 0.8759)	0.9700	0.0013	(0.9675; 0.9725)	(0.9666; 0.9739)
700	0.8649	0.0041	(0.8568; 0.8729)	(0.8547; 0.8745)	0.9705	0.0011	(0.9685; 0.9726)	(0.9675; 0.9729)
1000	0.8692	0.0032	(0.8629; 0.8755)	(0.8612; 0.8792)	0.9704	0.0009	(0.9685; 0.9721)	(0.9683; 0.9728)

Table 2: PM and HPD intervals for $A_S(\infty)$ and $A_P(\infty)$

(n = 40; Jeffreys' prior is assumed for the hyperparameters)

		$A_S(\infty)$			$A_P(\infty)$		
		PM	**st.dev.**	**95% HPD**	**PM**	**st.dev.**	**95% HPD**
$\lambda = \lambda_c = 0.1$	$\lambda_h = 0.07$	0.8212	0.0218	(0.7746; 0.8616)	0.9567	0.0062	(0.9440; 0.9682)
	$\lambda_h = 0.1$	0.7953	0.0253	(0.7424; 0.8408)	0.9508	0.0070	(0.9360; 0.9637)
$\mu = \mu_c = \mu_h = 3$	$\lambda_h = 0.4$	0.5908	0.0469	(0.5025; 0.6808)	0.8940	0.0162	(0.8592; 0.9241)
$\lambda = 0.1$	$\mu = \mu_c = \mu_h = 1$	0.6112	0.0374	(0.5401; 0.6859)	0.9084	0.0118	(0.8843; 0.9295)
	$\mu = \mu_c = \mu_h = 3$	0.8293	0.0209	(0.7862; 0.8677)	0.9644	0.0055	(0.9523; 0.9740)
$\lambda_c = \lambda_h = 0.07$	$\mu = \mu_c = \mu_h = 5$	0.8907	0.0155	(0.8573; 0.9188)	0.9778	0.0035	(0.9703; 0.9841)

Table 3: PM and HPD intervals for $A_S(\infty)$ and $A_P(\infty)$

(Two parameter gamma prior; Standard gamma prior and Jeffreys' prior assumed)

n	* Two parameter gamma prior				Standard gamma prior		Jeffreys' prior	
	$(\nu_1;\gamma_1) = (0.125;1.25)$; $(\nu_2;\gamma_2) = (1.225;17.5)$				$\nu_1 = 0.1$; $\nu_2 = \nu_3 = 0.07$			
	$(\nu_4;\gamma_4) = (\nu_5;\gamma_5) = (\nu_6;\gamma_6) = (10;3.3333)$				$\nu_4 = \nu_5 = \nu_6 = 3$			
	$(\nu_3;\gamma_3) = (0.49;7)$		$(\nu_3;\gamma_3) = (2.45;35)$					
	PM	**95% HPD**	**PM**	**95% HPD**	**PM**	**95% HPD**	**PM**	**95% HPD**
10	0.8000	(0.7104;0.8801)	0.8109	(0.7207; 0.8827)	0.7772	(0.6556; 0.8722)	0.7604	(0.6292; 0.8608)
20	0.8391	(0.7849; 0.8846)	0.8412	(0.7825; 0.8858)	0.8337	(0.7750; 0.8839)	0.8301	(0.7699; 0.8819)
40	0.8318	(0.7872; 0.8718)	0.8359	(0.7925; 0.8720)	0.8278	(0.7832; 0.8663)	0.8293	(0.7862; 0.8677)
100	0.8490	(0.8205; 0.8732)	0.8494	(0.8203; 0.8721)	0.8482	(0.8215; 0.8725)	0.8474	(0.8211; 0.8714)
500	0.8629	(0.8517; 0.8726)	0.8631	(0.8525; 0.8722)	0.8629	(0.8525; 0.8735)	0.8628	(0.8519; 0.8727)

Table 4a: PM and HPD intervals for $A_S(\infty)$

n	* Two parameter gamma prior				Standard gamma prior		Jeffreys' prior	
	$(\nu_1;\gamma_1) = (0.125;1.25)$; $(\nu_2;\gamma_2) = (1.225;17.5)$				$\nu_1 = 0.1$; $\nu_2 = \nu_3 = 0.07$			
	$(\nu_4;\gamma_4) = (\nu_5;\gamma_5) = (\nu_6;\gamma_6) = (10;3.3333)$				$\nu_4 = \nu_5 = \nu_6 = 3$			
	$(\nu_3;\gamma_3) = (0.49;7)$		$(\nu_3;\gamma_3) = (2.45;35)$					
	PM	**95% HPD**	**PM**	**95% HPD**	**PM**	**95% HPD**	**PM**	**95% HPD**
100	0.9613	(0.9401; 0.9782)	0.9630	(0.9403; 0.9793)	0.9567	(0.9223; 0.9777)	0.9538	(0.9203; 0.9768)
20	0.9646	(0.9511; 0.9781)	0.9649	(0.9492; 0.9774)	0.9636	(0.9451; 0.9771)	0.9635	(0.9461; 0.9773)
40	0.9645	(0.9529; 0.9729)	0.9646	(0.9532; 0.9745)	0.9639	(0.9527; 0.9732)	0.9644	(0.9523; 0.9739)
100	0.9690	(0.9630; 0.9748)	0.9689	(0.9632; 0.9746)	0.9687	(0.9621; 0.9741)	0.9688	(0.9625; 0.9745)
500	0.9701	(0.9670; 0.9725)	0.9701	(0.9672; 0.9728)	0.9699	(0.9660; 0.9725)	0.9700	(0.9675; 0.9725)

*Assumed prior information for two parameter gamma

Values of hyperparameters of λ_h	
$\omega_3 = 0.07$; $\sigma_3^2 = 0.01$	$\nu_3 = 0.49$; $\gamma_3 = 7$
$\omega_3 = 0.07$; $\sigma_3^2 = 0.002$	$\nu_3 = 2.45$; $\gamma_3 = 35$

and

Values of hyperparameters of λ, λ_c, μ, μ_c and μ_h		
$\omega_1 = 0.1$;	$\sigma_1^2 = 0.08$	$\nu_1 = 0.125$; $\gamma_1 = 1.25$
$\omega_2 = 0.07$;	$\sigma_2^2 = 0.004$	$\nu_2 = 1.225$; $\gamma_2 = 17.5$
$\omega_4 = \omega_5 = \omega_6 = 3$	$\sigma_4^2 = \sigma_5^2 = \sigma_6^2 = 0.9$	$\nu_4 = \nu_5 = \nu_6 = 10$ $\gamma_4 = \gamma_5 = \gamma_6 = 3.3333$

Table 4b: PM and HPD intervals for $A_p(\infty)$

(Beta prior assumed)

	$A_S(\infty)$				$A_P(\infty)$			
	$(m_3; r_3) = (5.313;76.9)$		$(m_3; r_3) = (1.1186;16.98)$		$(m_3; r_3) = (5.313;76.9)$		$(m_3; r_3) = (1.1186;16.98)$	
n	**PM**	**95% HPD**	**PM**	**95% HPD**	**PM**	**95% HPD**	**PM**	**95% HPD**
10	0.8157	(0.7330;0.8771)	0.8004	(0.7022; 0.8745)	0.9640	(0.9417; 0.9797)	0.9610	(0.9331; 0.9804)
20	0.8412	(0.7838; 0.8910)	0.8337	(0.7716; 0.8855)	0.9651	(0.9491; 0.9768)	0.9638	(0.9485; 0.9757)
40	0.8313	(0.7856; 0.8676)	0.8290	(0.7788; 0.8679)	0.9641	(0.9532; 0.9733)	0.9635	(0.9518; 0.9726)
100	0.8484	(0.8221; 0.8726)	0.8491	(0.8220; 0.8737)	0.9688	(0.9629; 0.9745)	0.9690	(0.9621; 0.9747)
500	0.8630	(0.8520; 0.8733)	0.8631	(0.8528; 0.8734)	0.9701	(0.9675; 0.9726)	0.9701	(0.9676; 0.9727)

Values of hyperparameters of λ_h	
$\omega_3 = 0.07$; $\sigma_3^2 = 0.01$	$m_3 = 1.1186$; $r_3 = 16.98$
$\omega_3 = 0.07$; $\sigma_3^2 = 0.001$	$m_3 = 5.313$; $r_3 = 76.9$

and

Values of hyperparameters of λ, λ_c, μ, μ_c and μ_h		
$\omega_1 = 0.1$;	$\sigma_1^2 = 0.09$	$m_1 = 0.22222$; $r_1 = 3.2222$
$\omega_2 = 0.07$;	$\sigma_2^2 = 0.008$	$m_2 = 0.72538$; $r_2 = 11.363$
$\omega_4 = \omega_5 = \omega_6 = 3$	$\sigma_4^2 = \sigma_5^2 = \sigma_6^2 = 1.5$	$m_4 = m_5 = m_6 = 27$ $r_4 = r_5 = r_6 = 10$

Table 5: PM and HPD intervals for $A_S(\infty)$ and $A_P(\infty)$

6. CONCLUSIONS

A two component series and parallel system with common-cause shock failures and human error has been studied. The steady-state availability for these systems are obtained as a system measure. Using the classical estimation (Table 1) and the Bayesian estimation (Table 2, 3, 4a, 4b and 5 and Figure 1), the confidence limits for the steady-state availability has been obtained. From these two procedures, it can be concluded that, as the sample size increases the steady-state availability increases for different parameters. From all the intervals it is clear that the system availability for the parallel configuration is higher than the system availability for the series configuration. One can influence the human error factor by reducing the failure rate λ_h through specialized training. Similar options can be chosen for the other hyperparameters. Depending on the prior belief one can achieve good results for small sample sizes as well. The simulated samples are shown in Table 6. The results obtained have justified the models we studied.

			$\overline{U}_1$	$\overline{U}_2$	$\overline{U}_3$	$\overline{U}_4$	$\overline{U}_5$	$\overline{U}_6$
$\lambda = \lambda_c = 0.1;\ \lambda_h = 0.1$	$\mu = \mu_c = \mu_h = 3$	$n = 40$	9.5531	9.3588	6.6019	0.4219	0.2965	0.3419
$\lambda = \lambda_c = 0.1;\ \lambda_h = 0.4$	$\mu = \mu_c = \mu_h = 3$	$n = 40$	9.5531	9.3588	1.6505	0.4219	0.2965	0.3419
$\lambda = \lambda_c = 0.1;\ \lambda_h = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 40$	9.5531	9.3588	9.4312	0.4219	0.2965	0.3419
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 40$	9.5531	13.3696	9.4312	0.4219	0.2965	0.3419
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 5$	$n = 40$	9.5531	13.3696	9.4312	0.2531	0.1779	0.2051
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 1$	$n = 40$	9.5531	13.3696	9.4312	1.2657	0.8895	1.0256
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 40$	9.8909	17.4877	7.6879	0.5142	0.3412	0.5007
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 20$	10.9477	14.4626	8.5106	0.3998	0.3318	0.3405
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 100$	8.5011	14.3940	13.1298	0.3763	0.2956	0.3430
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 500$	9.5615	14.3971	13.9151	0.3407	0.3174	0.3665
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n = 700$	9.7978	14.4616	13.4949	0.3386	0.3141	0.3580
$\lambda = 0.1; \lambda = \lambda_c = 0.07$	$\mu = \mu_c = \mu_h = 3$	$n= 1000$	9.9816	14.3897	13.7572	0.3292	0.3224	0.3475

Table 6: Sample means

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